Total No. of Printed Pages:02

SUBJECT CODE NO- NEPHR-01-2025
FACULTY OF SCIENCE AND TECHNOLOGY
EXAMINATION WINTER 2025
M.SC (FIRST YEAR) (SEM-I)
COMMON PAPER
SVECRM-401-RESEARCH METHODOLOGY(COMPULSORY)

[Time: 3:00 Hours] **[Max.Marks:45]**

"Please check whether you have got the right question paper."

N.B.
1. Question No. 1 is Compulsory.
2. Solve any TWO questions from Question No. 2 to 5.
3. Calculator and log table allowed.

Q.1 Write notes on: **5X3=15**
1. Research objectives
2. Features of good research designing
3. Editing processing operations
4. statistical measures in research
5. Variables

Q.2 1. Describe various steps involved in research. **08**
2. Explain types of research hypothesis. **07**

Q.3 1. Explain meaning and need of good research designing. **08**
2. Describe descriptive and fundamental types of research. **07**

Q.4 1. Calculate, mean, median and mode of the following data. **08**

Class Interval (CI)	Frequency (F)
50-54	2
45-49	5
40-44	8
35-39	7
30-34	10
25-29	5
20-24	9
15-19	2
10-14	1
5-9	1

2. Describe observation method for collection of primary data. **07**

Q.5 1. calculate chi square (x^2) value of the following data. **08**

Excellent	Average	Poor	Total
58	32	30	120

2. Explain in detail case study. **07**

This question paper contains **4** printed pages]

NEPHR—483—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

SMATE-401(D)

(Theory of Probability)

(Wednesday, 24-12-2025) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—60

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt the following : -15

(*a*) Define mathematical and statistical probability.

P.T.O.

(*b*) Define a one-dimensional random variable on a σ-field B of subsets of a sample spaces S.

(*c*) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

(*d*) If X is normally distributed and the mean of X is 12 and the standard deviation is 4, find the probability of $X \leq 20$.

(*e*) If X is a random variable and a is a constant, then with usual notations, proev that :

$$V(a. X + b)\, a^2\, V(X).$$

2. Attempt the following : 15

State and prove the addition theorem of probability to n events A_1, A_2 ..., A_n. An integer is chosen at random from 1 to 200. What is the probability that the integeris divisible by 6 or 7 ?

3. Attempt the following : 15

Define moments generating function of a discrete and continuous random variable. Obtain the expression for the moments generating function of a discrete random variable about a point X = a. Give an example to prove that a random variable may have no moments although its m.g.f. exists.

4. Attempt the following : 15

Define Poisson distribution. State the conditions under which Poisson distribution is applied. Obtain the expressions for first four moments about origin of the Poisson distribution. Prove that the mean and variance of the Poisson distribution are both equal to the parameter of the distribution.

5. Answer the following : 15

The local authorities in a city install 10000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail :

(*i*) in the first 800 burning hours

(*ii*) between 800 and 1200 burning hours

After what period of burning hours would you expect that :

(*a*) 10% of the lamps would fail

(*b*) 10% of the lamps would be still burnings ?

6. Answer any *three* of the following : 15

(*a*) A box contains 6 red, 4 white and 5 black ballls. A person draws 4 balls from the bag at random. Find the probability that among the balls drawn, there is at least one ball of each colour.

P.T.O.

(*b*) Let the random variable X assume the value *r* with the probability law :

$$P(X = r) = q^{r-1} p; \quad r = 1, 2, 3 ......$$

Find the moments generating function of X.

(*c*) The mean and variance of a binomial variate are 4 and 4/3 respectively. Find $P(X \geq 1)$.

(*d*) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the arithmetic mean and the standard deviation of the distribution ?

Total No. of Printed Pages:01

SUBJECT CODE NO- NEPHR-480-2025
FACULTY OF SCIENCE AND TECHNOLOGY
EXAMINATION WINTER 2025
M.SC. (FIRST YEAR) (SEM-I)
MATHEMATICS
ORDINARY DIFFERENTIAL EQUATIONS

[Time: 2:30 Hours] **[Max.Marks:60]**

"Please check whether you have got the right question paper."

N.B.
1. All questions carry equal marks.
2. Q.No. 1 is compulsory.
3. Answer any three from Q.No. 2 to Q.No. 6.
4. Figures to the right indicate full marks.

Q.1 Answer the following **15**

i. Solve $3y'' + 2y' = 0$

ii. Find the second independent solution of $x^2y'' - xy' + y = 0$ if $\phi_1(x) = x(x > 0)$ is one of the solution.

iii. Solve $x^2y''' + 2x^2y'' - xy' + y = 0$ for $x > 0$.

iv. Find all real valued solutions of $y' = \frac{e^{x-y}}{1+e^x}$

v. Write Lipschitz condition.

Q.2 Prove that the two solutions $\phi_1(x), \phi_2(x)$ of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I. **15**

Q.3 Derive two solutions of Legendre equation. **15**

Q.4 Prove that the solution of $x^2y'' + a(x)xy' + b(x)y = 0$ is given by$\phi_i(x) = | x |^{r_i} \sum_{k=0}^{\infty} C_k x^k$, where $i = 1,2$ and a, b have convergent power series expansions. **15**

Q.5 Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I. **15**

Q.6 Answer any three the following **15**

a) Solve $4y'' - y = e^x$

b) Compute the Wronskian if characteristics polynomial of $L(y) = 0$ is$(r - r_1)^3$.

c) Solve $y'' + 4y' = 0$

d) Prove that the solution of $y' = xy, y(0) = 1$ is $e^{x^2/2}$

This question paper contains **3** printed pages]

NEPHR—481—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020 Pattern)

MATHEMATICS

Paper SMATE-401-(B)

(Discrete Mathematics)

(Wednesday, 24-12-2025) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—60

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 15

(*a*) State the principle of duality.

P.T.O.

(*b*) Define subgraph of a graph with suitable example.

(*c*) Define a fundamental cut-set of a spanning tree in a connected graph.

(*d*) Define a simple diagraph with suitable example.

(*e*) Define a complete diagraph with suitable example.

2. Answer the following : 15

Define the universal upper and lower bounds in a lattice with a suitable example. Let $(A, \leq)$ be a lattice with universal upper and lower bounds 1 and 0. Then for every a in $(A, \leq)$ prove that :

$$a \vee 1 = 1 \quad a \wedge 1 = a$$
$$a \vee 0 = a \quad a \wedge 0 = 0$$

3. Answer the following : 15

Define a path in graph with suitable example. Prove that if a graph (connected or disconnected) has exactly two vertices of odd degree, then there must be a path joining these two vertices.

4. Answer the following : 15

(*i*) Define a tree with suitable examples. Prove that a tree with n vertices has $n - 1$ edges.

(*ii*) Prove that a connected graph with n vertices and $n - 1$ edges is a tree.

5. Answer the following : 15

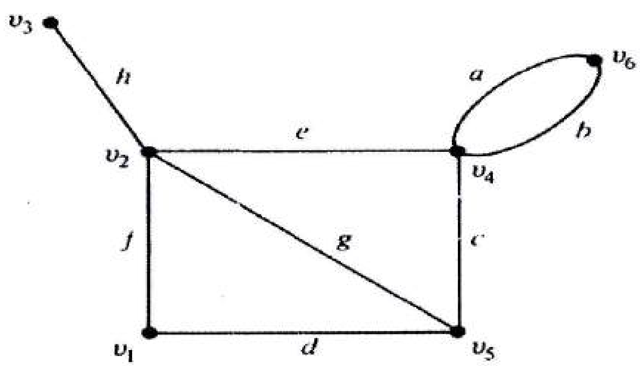
If B is a circuit matrix of a connected graph G with e edges and n vertices, then prove that rank of B = $e - n + 1$.

6. Answer of the following : 15

(*a*) Write a short note on travelling salesman problem

(*b*) Prove that the edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

(*c*) Obtain a circuit matrix of the following graph :



Total No. of Printed Pages:01

SUBJECT CODE NO- NEPHR-482-2025

FACULTY OF SCIENCE AND TECHNOLOGY

EXAMINATION WINTER 2025

M.SC. (FIRST YEAR) (SEM-I)

MATHEMATICS

DYNAMICS & CONTINUUM MECHANICS-I

[Time: 2:30 Hours] **[Max.Marks:60]**

"Please check whether you have got the right question paper."

N.B.
1. All questions carry equal marks.
2. Q.No. 1 is compulsory.
3. Answer any three from Q.No. 2 to Q.No. 6.
4. Figures to the right indicate full marks.

Q.1 Answer the following **15**
1. Explain vector moment about a point.
2. Define mass, moment and force.
3. Explain kinetic energy of rigid body.
4. Define moment and product of inertia.
5. Define angular momentum of a rigid body about fixed axes.

Q.2 Derive an expression for velocity and acceleration for moving axis. **15**

Q.3 Describe principle of conservation of energy established for a single particle. **15**

Q.4 Derive an expression for moving origin. **15**

Q.5 Illustrate coplanar distribution. **15**

Q.6 Answer any three of the following **15**

a) Explain velocity and acceleration of a particle along a curve

b) Define conservative forces and illustrate earth's gravitational field.

c) Prove that kinetic energy of a system of particle in centroid is $\frac{1}{2}M\dot{r}^2+\Sigma \mathrm{M}\dot{r}^2$

d) Prove that kinetic energy of a rigid body rotating about a fixed point is $\frac{1}{2}\omega^2 I$.

This question paper contains **4** printed pages]

NEPHR—177—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

Paper SMATC-402

(Real Analysis)

(Wednesday, 17-12-2025) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—60

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following questions : 15

(*a*) State fundamental theorem of calculus.

P.T.O.

(*b*) Define pointwise convergence of sequence of functions.

(*c*) State the Stone-Weierstrass theorem.

(*d*) Let $f : R^2 \to R^2$ be define by the equation :

$$f(x, y) = (e^x \cos y, e^x \sin y)$$

Calculate D*f* and $|Df|$.

(*e*) Does the integrability of $|f|$ implies that of f ? Justify your answer.

2. Answer the following questions : 15

If P* is a refinement of P, then prove that :

$$L(P, f, \alpha) \leq L(P^*, f, \alpha) \text{ and}$$
$$U(P^*, f, \alpha) \leq U(P, f, \alpha)$$

Also, suppose $f \geq 0, f$ is continuous on $[a, b]$ and $\int_a^b f(x)\,dx = 0$. Show that $f(x) = 0$ for all $x \in [a, b]$.

3. Answer the following questions : 15

State and prove the Weierstrass M-test for uniform convergence for sequence of function. Also, for :

$$m = 1, 2, 3, \ldots., n = 1, 2, \ldots.., \text{let } S_{mm} = \frac{m}{m+n}$$

show that :

$$\lim_{n \to \infty} \lim_{m \to \infty} S_{mm} \neq \lim_{m \to \infty} \lim_{n \to \infty} S_{mn}$$

4. Answer the following questions : 15

Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$ ($|x| < R$). Then prove that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\varepsilon, R-\varepsilon]$, no matter which $\varepsilon > 0$ is chosen. The function f is continuous and differentiable on $(-R, R)$, and

$$f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1} (|x| < R).$$

5. Answer the following questions : 15

Let, $A \subseteq R^m$, let $f : A \to R^n$, if f is differentiable at $\bar{a}$, then prove that all the directional derivative of f at $\bar{a}$ exist and $f'(\bar{a}, \bar{u}) = Df(\bar{a}).\bar{u}$ also, given $f : R^5 \to R^2$ of class C^1. Let $\bar{a}$ = (1, 2, –1, +3, 0). Suppose that $f(\bar{a}) = \bar{0}$ and :

$$Df(\bar{a}) = \begin{bmatrix} 1 & 3 & 1 & -1 & 2 \\ 0 & 0 & 1 & 2 & -4 \end{bmatrix}$$

(*i*) Show there is a function $f : B \to R^2$ of class C^1 defined on an open set B of R^3 such that :

$$f(x_1, g_1(x), g_2(x), x_2 x_3) = \bar{0} \text{ for } \bar{x} = (x_1, x_2, x_3) \in B \ \& \ g(1, 3, 0) = (2, -1)$$

(*ii*) Find D*g* (1, 3, 0).

6. Attempt any *three* of the following : 15

(*a*) Write a short note on rectifiable curve.

P.T.O.

(*b*) Let :

$$f_n(x) = n^2 x\,(1 - x^2)^n,\ 0 \le x \le 1$$

and n = 1, 2, 3, ..., show that :

$$\lim_{n\to\infty}\left[\int_0^1 f_n(x)\,dx\right] \ne \int_0^1\left[\lim_{n\to\infty} f_n(x)\right]dx.$$

(*c*) Does the every member of an equicontinuous family is uniformly continuous ? Justify your answer.

(*d*) Define $f : \mathrm{R}^2 \to \mathrm{R}$ by setting $f(\bar{0}) = \bar{0}$ and $f(x, y) = \dfrac{x^2}{x^4 + y^2}$ if $f(x, y) \ne \bar{0}$. Show that all directional derivative of f exist to $\bar{0}$ but that f is not differentiable at $\bar{0}$.

Total No. of Printed Pages:2

SUBJECT CODE NO:- NEPHR-128-2025
FACULTY OF SCIENCE & TECHNOLOGY
EXAMINATION WINTER 2025
M.Sc. (FIRST YEAR) (SEM –II)
MATHEMATICS
SMATC – 452 MEASURES AND INTEGRATION THEORY

[Time: 2:00 Hours] **[Max.Marks:60]**

"Please check whether you have got the right question paper."

N.B.
i) All questions carry equal marks.
ii) Q. No. 1 is Compulsory.
iii) Answer any three from Q. No. 2 to Q. No. 6.
iv) Figures to the right indicate full marks.

Q. 1 Answer the following **15**

a) Show that every countable set has measure zero.

b) Show that, if $f \in BV[a,b]$, then f is bounded on $[a,b]$.

c) Prove that, every measure is a signed measure.

d) Show that, a countable union of sets positive with respect to a signed measure ν is positive set.

e) Define outer measure of set.

Q. 2 Answer the following **15**

Prove that, the class M is a σ-algebra. Also, Show that $F \in \mu$ and $m^*(F \Delta G) = 0$, then G is measurable.

Q. 3 Answer the following **15**

Let $f \in BV[a,b]$, then prove that: $f(b) - f(a) = P - N$ and $T = P + N$, where all variations being in the finite interval $[a,b]$. Also, Let f be defined by $f(x) = | x |$. Find the four derivative at $x = 0$.

Q. 4 Answer the following **15**

Let $\{A_i\}$ be a sequence in a ring R, then prove that there is a sequence $\{B_i\}$ of disjoint sets of R such that $B_i \subseteq A_i$ for each i and $\cup_{i=1}^{N} A_i = \cup_{i=1}^{N} B_i$
For each N, so that $\cup_{i=1}^{\infty} A_i = \cup_{i=1}^{\infty} B_i$
Also, Show that, $\mathrm{H(R)} = \{\mathrm{E} ! \ \mathrm{E} \subseteq \cup_{n=1}^{\infty} \mathrm{E_n}, \mathrm{E_n} \in \mathrm{R}\}$

Q. 5 Answer the following **15**

State and prove Hahn decomposition theorem. Also, Show that if:$\phi(E) = \int_E f d\mu$, where$\int f d\mu$ is defined, then ϕ is a signed measure.

Q. 6 Attempt any three of the following **15**

a) Show that, the constant functions are measurable.

b) Show that, if μ is a σ-finite measure on **R**, then the extension $\bar{\mu}$ of to $\mu\ to\ s^*$is also σ-finite.

c) If $f \in L(a, b)$ then prove that and $f \in BV[a, b]$.

d) Show that $v^+ = \frac{1}{2}(v+| v |)$,$v^- = \frac{1}{2}(| v | -v)$, provided v is finite valued

Total No. of Printed Pages:2

SUBJECT CODE NO:- NEPHR-128-2025
FACULTY OF SCIENCE & TECHNOLOGY
EXAMINATION WINTER 2025
M.Sc. (FIRST YEAR) (SEM –II)
MATHEMATICS
SMATC – 452 MEASURES AND INTEGRATION THEORY

[Time: 2:00 Hours] **[Max.Marks:60]**

"Please check whether you have got the right question paper."

N.B.
i) All questions carry equal marks.
ii) Q. No. 1 is Compulsory.
iii) Answer any three from Q. No. 2 to Q. No. 6.
iv) Figures to the right indicate full marks.

Q. 1 Answer the following **15**

a) Show that every countable set has measure zero.

b) Show that, if $f \in BV[a,b]$, then f is bounded on $[a,b]$.

c) Prove that, every measure is a signed measure.

d) Show that, a countable union of sets positive with respect to a signed measure v is positive set.

e) Define outer measure of set.

Q. 2 Answer the following **15**

Prove that, the class M is a σ-algebra. Also, Show that $F \in \mu$ and $m^*(F\Delta G) = 0$, then G is measurable.

Q. 3 Answer the following **15**

Let $f \in BV[a,b]$, then prove that: $f(b) - f(a) = P - N$ and $T = P + N$, where all variations being in the finite interval $[a,b]$. Also, Let f be defined by $f(x) =| x |$. Find the four derivative at $x = 0$.

Q. 4 Answer the following **15**

Let $\{A_i\}$ be a sequence in a ring R, then prove that there is a sequence $\{B_i\}$ of disjoint sets of R such that $B_i \subseteq A_i$ for each i and $\cup_{i=1}^{N} A_i = \cup_{i=1}^{N} B_i$
For each N, so that $\cup_{i=1}^{\infty} A_i = \cup_{i=1}^{\infty} B_i$
Also, Show that, $\mathrm{H(R)} = \{\mathrm{E}!\ \mathrm{E} \subseteq \cup_{n=1}^{\infty} \mathrm{E_n}, \mathrm{E_n} \in \mathrm{R}\}$

Q. 5 Answer the following **15**

State and prove Hahn decomposition theorem. Also, Show that if:$\phi(E) = \int_E f d\mu$, where$\int f d\mu$ is defined, then ϕ is a signed measure.

Q. 6 Attempt any three of the following **15**

a) Show that, the constant functions are measurable.

b) Show that, if μ is a σ-finite measure on **R**, then the extension $\bar{\mu}$ of to μ *to* $s^*$is also σ-finite.

c) If $f \in L(a, b)$ then prove that and $f \in BV[a, b]$.

d) Show that $v^+ = \frac{1}{2}(v + | v |)$,$v^- = \frac{1}{2}(| v | - v)$, provided v is finite valued

Total No. of Printed Pages:01

SUBJECT CODE NO- NEPHR-400-2025
FACULTY OF SCIENCE AND TECHNOLOGY
EXAMINATION WINTER 2025
M.SC. (FIRST YEAR) (SEM-II)
MATHEMATICS
PARTIAL DIFFERENTIAL EQUATIONS

[Time: 2:30 Hours] **[Max.Marks:60]**

"Please check whether you have got the right question paper."

N.B.
1. All questions carry equal marks.
2. Q.No. 1 is compulsory.
3. Answer any three from Q.No. 2 to Q.No. 6.
4. Figures to the right indicate full marks.

Q.1) Answer the following **15**
- i) Eliminate arbitrary function $F(xy, x + y - z) = 0$ and find corresponding p.d.e.
- ii) Explain analytic expression for the Monge cone at (x_0, y_0, z_0)
- iii) Write the Neumann and Robin boundary value problem
- iv) State Harnack theorem.
- v) Solve $p + q = z$

Q.2) If $\bar{X} \cdot curl\ \bar{X} = 0$ where $X = P\bar{\imath} + Q\bar{\jmath} + R\bar{\bar{k}}$ and μ is an arbitrary differentiable function of x,y and z then prove that $\mu\bar{X} \cdot curl(\mu\bar{X}) = 0$ **15**

Q.3) Find complete integral of $2(z + xp + yq) = yp^2$ by Charpit's method. **15**

Q.4) Derive canonical form for hyperbolic and elliptic type of pde. **15**

Q.5) Show that the solution for the Dirichlet problem for a circle is given by the Poisson Integral Formula. **15**

Q 6) Answer Any Three of the following. **15**
- a) Find the general integral of $yzp + xzq = xy$
- b) Write a note on classification of Integrals.
- c) Reduce into canonical forms and solve $U_{xx} + (x^2)U_{yy} = 0$
- d) Show that the solution of the Dirichlet problem if it exists is unique.

This question paper contains 4 printed pages]

NEPHR—401—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

SMATE-451B

(Combinatorics)

(Tuesday, 23-12-2025) **Time : 10.00 a.m. to 12:30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—60

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt the following questions : 15

(*a*) How many different sequences of heads and tails are possible if a coin is flipped 100 times ? Using the fact that $2^{10} = 1024 \approx 1000 = 10^3$, give your answer in terms of an approximate power of 10.

P.T.O.

(*b*) How many ways are there to select 25 toys from seven types of toys with between two ands six of each type ?

(*c*) Find a recurrence relation for number of ways to arrange n distinct objects on a row.

(*d*) How many distinct arrangements of the letters S, T, O, P can be formed with repetitions of the letters allowed ?

(*e*) Two dice are rolled, one green and one red. Each die has faces numbered 1 through 6. How many different outcomes of this procedure are there ?

2. Attempt the following questions : 15

(*i*) If there are n objects, with r_1 of type 1, r_2 of type 2, ..., and r_m of type m, where $r_1 + r_2 + \ldots\ldots + r_m = n$, then prove that the number of arrangements of these n objects, denoted $P(n; r_1, r_2, \ldots. r_m)$ is given by :

$$P(n; r_1, r_2 \ldots, r_m) = \binom{n}{r_1}\binom{n-r_1}{r_2}\binom{n-r_1-r_2}{r_3} \ldots. \binom{n-r_1-r_2-\ldots.-r_{m-1}}{r_m}$$

$$= \frac{n!}{r_1!\, r_2! \ldots r_m!}$$

(*ii*) Prove that the number of selections with repetition of r objects chosen from n types of objects is given by $C(r + n - 1, r)$

3. Answer the following question : 15

Use a generating function to model the problem of counting all selections of six objects chosen from three types of objects with repetition of up to four objects of each type. Also model the problem with unlimited repetition.

4. Attempt the following question : 15

A bank pays 4 percent interest each year on money in savings accounts. Find recurrence relations for the amounts of money a gnome would have after n years if it follows the investment strategies of :

(*a*) Investing Rs. 1,000 and leaving it in the bank for n years.

(*b*) Investing Rs. 100 at the end of each year.

5. Attempt the following question : 15

Let $A_1, A_2, \ldots\ldots, A_n$ be n sets in the universal set U of N elements. Let S_k denote the sum of the sizes of all k-tuple intersection of the A_is. Then prove that :

$$N(\bar{A}_1 . \bar{A}_2 \ldots \bar{A}_n) = N - S_1 + S_2 - S_3 + \ldots + (-1)^k S_k + \ldots . + (-1)^n S_n$$

P.T.O.

6. Answer any *three* of the following : 15

(*a*) How many wasys are there to distribute seven different books among 15 children if not child gets more than one book ?

(*b*) Find the generating function for a_r, then number of ways to select r balls from three green, three white and three gold balls.

(*c*) Solve the following recurrence relations assuming that n is a power of 2 (leaving a constant A to be determined) :

$$a_n = 2a_{n/2} + 5$$

(*d*) If a school has 100 students with 50 students taking French, 40 students taking Latin, and 20 students taking both languages, how many students take no language.

This question paper contains 4 printed pages]

NEPHR—41—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020 Pattern)

MATHEMATICS

Paper SMATC-451

(Linear Algebra)

(Saturday, 13-12-2025) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—60

N.B. :— (*i*) Question No. **1** is compulsory.

(*ii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iii*) Figures to the right indicate full marks.

1. Attempt the following questions (**3** marks each) 15

(*a*) Prove that intersection of two subspaces H and K of vector space V is also subspace of V.

P.T.O.

(*b*) Prove that :

$$W_1 = \{(a_1, a_2, a_3) \in R^3 : a_1 + 3a_2 = 0 \text{ and } a_3 + a_2 = 0\}$$

is the subspace of R3.

(*c*) If V and W are vector spaces of same dimension n, and let : T : V $\rightarrow$ W be linear such that N(T) = {0}, then prove that T is one-to-one.

(*d*) If X and Y are two vectors of inner product space V, then prove that :

$$\|X + Y\|^2 + \|X - Y\|^2 = 2\ \|X\|^2 + 2\ \|Y\|^2$$

(*e*) If M be a square lower triangular matrix with non-zero diagonal entries, then prove that the rows of M are linearly independent.

2. Attempt the following questions : 15

(*a*) Let V be a vector space, and let $S1 \subseteq S2 \subseteq V$. If S2 is linearly independent, then prove that S2 is linearly independent. 7

(*b*) If V be the finite dimensional vector space over F and W be any subspace of V, then prove that W is also finite dimensional. 8

3. Attempt the following questions : 15

(*a*) Let V be a finite dimensional vector space over F and if $T \varepsilon L(V)$ is an T is onto linear operator, then prove that rank(T) = dim (V). 7

(*b*) Let V and W be vector spaces and let $T : V \rightarrow W$ be linear. If V is finite dimensional, then prove that nullity(T) + rank(T) = dim(V). 8

4. Attempt the following questions : 15

(*a*) Let $Ax = b$ be a system of n linear equations in n unknowns. If A is invertible, then prove that the system has exactly one solution, namely, $A^{-1}b$. 7

(*b*) Let $T : V \rightarrow W$ and $U : W \rightarrow Z$ be linear transformations on finite-dimensional vector spaces V, W and Z, then prove that $\text{rank}(UT) \leq \text{rank}(T)$. 8

5. Attempt the following questions : 15

(*a*) Let V be an inner product space over F and if XY are any two vectors of V, then prove that : 7

$$|(X, Y)| \leq \|X\| \|Y\|$$

(*b*) If V be an inner product space, and let S be an orthonormal subset of V consisting of non-zero vectors, then prove that S is linearly independent. 8

P.T.O.

6. Attempt any *three* of the following (**5** marks each) : 15

(*a*) If W is a subspace of a finite-dimensional vector space V, then prove that any basis for W can be extended to a basis for V.

(*b*) Let V and W be vector spaces over the same field F. If T : V $\to$ W and U : V $\to$ W are linear mapping, then prove that U + T : V $\to$ W is linear.

(*c*) If T is a linear operator on a vector space V, then prove that N(T) is invariant under T.

(*d*) Let B = $\{w_1 = (1, 0, 0, 0), w_2 = (1, 2, 0, 0), w_3 = (1, 2, 3, 4\}$ is linearly independent, then use the Gram-Schmidt process to compute the orthogonal vectors v_1, v_2 and v_3, and then normalize these.

This question paper contains **3** printed pages]

NEPHR—83—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (SE) (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

SMATC-502

(Functional Analysis)

(Monday, 15-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time— 3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following questions : 20

(*a*) If N is normed linear space and $x_0 \neq 0 \in N$, prove that, there exist a functional f_0 in N* such that :

$$f_0(x_0) = ||x_o|| \text{ and } ||f_0|| = 1.$$

P.T.O.

(*b*) State and prove parallelogram law.

(*c*) For an arbitrary operator T on H, prove that, TT* & T*T are self-adjoint.

(*d*) If T is normal operator, then show that x is an eigen vector of T if and only if x is eigen vector of T*

2. Answer the following questions : 20

(*a*) State and prove Hahn-Banach Theorem.

(*b*) State and prove the closed graph theorem.

3. Answer the following questions : 20

(*a*) State and prove Schwartz's inequality

(*b*) State and prove Bessel's inequality.

4. Answer the following questions : 20

(*a*) If T is an arbitrary continuous linear operator on Hilbert space H, then prove that T* is a continuous linear transformation on Hilbert space H.

(*b*) Define a unitary operator. Prove that an operator T on Hilbert space H is unitary if and only if T is an isometric isomorphism of H onto itself.

5. Answer the following questions : 20

(*a*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, ... \lambda_m$, be the distinct eigne values of T corresponding to eigen vectors $x_1, x_2 .... x_m$. Suppose $M_1, M_2, ... M_m$ are corresponding eigen space and $P_1, P_2, ... P_m$ are the projections on these eigen spaces.

If M_i's are pairwise orthogonal and spans H, then prove that, P_i's are pairwise orthogonal :

$$I = \sum_{i=1}^{m} P_i \text{ and } T = \sum_{i=1}^{m} \lambda_i P_i .$$

(*b*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, ... \lambda_m$, be the distinct eigne values of T corresponding to eigen vectors $x_1, x_2 .... x_m$. Suppose $M_1, M_2, ... M_m$ are corresponding eigen space and $P_1, P_2, ... P_m$ are the projections on these eigen spaces.

If T is normal operator on H, then prove that, M_i's are pairwise orthogonal.

6. Answer the following questions : 20

(*a*) If N is a non-zero normed linear space, then prove that, N is Banach space if and only if $S = \{x \in N \,|\, ||x|| = 1\}$ is complete.

(*b*) Define eigen space. If T is normal operator on H, then prove that, each M_i reduces T.

This question paper contains **3** printed pages]

NEPHR—178—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

Paper SMATC-503

(Analytical Number Theory)

(Wednesday, 17-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time— 3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Show that 41 divides $2^{20} - 1$.

P.T.O.

(*b*) Find the four primitive roots of 26.

(*c*) Find the values of the following Legendre symbols :

(19/23) and (–23/59).

(*d*) If $n \geq 1$, then prove that :

$$\log n = \sum_{d|n} \wedge(d).$$

2. Answer the following : 20

(*a*) State and prove Fermat's little theorem.

(*b*) Solve $18x \equiv 30 \pmod{42}$

3. Answer the following : 20

(*a*) If p is a prime number and $d \mid p - 1$, then prove that the congruence : $x^d - 1 \equiv 0 \pmod{p}$ has exactly d solutions.

(*b*) Find the order of integers 2, 3, 5 :

(*i*) Modulo 17

(*ii*) Modulo 19.

4. Answer the following : 20

(*a*) State and prove quadratic reciprocity law.

(*b*) Solve the quadratic congruence :

$$5x^2 + 6x + 2 \equiv 0 \pmod{13}$$

5. Answer the following ; 20

(*a*) Let f be multiplicative, then prove that, if f is completely multiplicative if and only if :

$$f^{-1}(n) = \mu(n) \,.\, f(n), \quad \text{for all } n \geq 1.$$

(*b*) Define Mangoldt function $\wedge(n)$. Find the values of Mangoldt function for $n = 1$ to 10.

6. Answer the following : 20

(*a*) Give an example to show that $a^2 \equiv b^2 (\bmod n)$ does not implies $a \equiv b (\bmod n)$. Also, verify that 3 is a primitive root of 7.

(*b*) Determine whether the quadratic congruence is solvable or not $x^2 \equiv -46 \ (\bmod 17)$ and also find all integer n such that $\phi(n) = n/2$.

This question paper contains **2** printed pages]

HR—277—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(New/CBCS Pattern)

MATHEMATICS

Paper XVII

(Fluid Mechanics-I)

(Monday, 22-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (1) Each question carries equal marks.

(2) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(A) Fluid flows through a pipe whose surface is the surface of revolution of the curve $y = a + \frac{kx^2}{a}$, where $-a \le x^2 \le a$ about the x-axis. If the liquid enters at the end $x = -a$ of the pipe with velocity v, then show that the time taken by liquid particles to traverse the entire length of pipe from $x = -a$ to $x = a$ is :

$$T = \frac{2a}{v(1+k)^2}\left[1 + \frac{2k}{3} + \frac{k^2}{5}\right].$$

P.T.O.

Or

(B) Define velocity potential. Derive an equation of continuity for steady incompressible fluid.

2. Attempt any *one* of the following : 15

(A) Prove that, in fluid region the pressure is same in all direction.

Or

(B) Explain the mechanism of Venturi tube.

3. Attempt any *one* of the following : 15

(A) State and prove Kelvin's theorem.

Or

(B) Explain the motion of accelerating sphere moving in fluid at rest at infinity.

4. Attempt any *one* of the following : 15

(A) Describe the irrotational motion of an incompressible liquid for which the complex potential is $w = ik \log Z$.

Or

(B) Discuss the flow due to uniform line doublet at origin of strength u. Its axis being along x-axis.

5. Attempt any *three* of the following : 15

(*i*) Derive an expression for local and particle rates of change.

(*ii*) Derive an expression for strength of vortex tube.

(*iii*) Derive Bernoulli's equation.

(*iv*) Write a note on line sink.

This question paper contains 4 printed pages]

NEPHR—300—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP-2020 Pattern)

MATHEMATICS

Paper SMATE-501(A)

(Integral Transforms)

(Monday, 22-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find the Laplace transform of cos at and sin at.

(*b*) Find the Mellin transform of $1/(1 + x)$.

(*c*) Find a Fourier integral representation of the function $f(x) = e^{-|x|}$, $-\infty < x < \infty$.

P.T.O.

(*d*) Define the Hankel transform of order zero of a function $f(r)$ in polar co-ordinates and its inversion formula.

2. Answer the following : 20

(*a*) If f is piecewise continuous on $t \geq 0$ and is $0\left(e^{c_0 t}\right)$, then prove that $f(t)$ has a Laplace transform $F(p)$ in the half-plane $\text{Re}(p) > c_0$. Moreover, prove that the Laplace transform integral converges both absolutely and uniformly for $\text{Re}(p) \geq c_2 > c_0$.

(*b*) Find the Laplace transform of :

$$f(t) = \int_0^t \left(u^2 - u + e^{-u}\right) du.$$

3. Answer the following : 20

(*a*) Using Laplace transform, solve the initial value problem :

$$y'' - 3y' + 2y = 4e^{2t}, \quad y(0) = -3, \quad y'(0) = 5.$$

(*b*) Given the heat conduction problem :

$$u_{xx} = a^{-2}u_t, \qquad 0 < x < \infty,\ t > 0$$

B.C. : $u_x(0, t) = -f(t), \qquad u(x, t) \to 0$ as $x \to \infty$,

I.C. : $u(x, 0) = 0, \qquad 0 < x < \infty$,

show that its solution can be expressed as :

$$u(x, t) = \frac{a}{\sqrt{\pi}} \int_0^t \frac{f(\tau)}{\sqrt{t - \tau}} \exp\left(\frac{-x^2}{4a^2(t - \tau)}\right) d\tau.$$

4. Answer the following : 20

(*a*) Define the Fourier transform of a function $f(t)$ and its inverse Fourier transform. If $\mathbf{F}\{f(t)\} = F(s)$ represents the Fourier transform of a function $f(t)$, prove that :

(*i*) $\mathbf{F}\{e^{iat} f(t)\} = \mathbf{F}(s + a)$

(*ii*) $\mathbf{F}\{f(t - a)\} = e^{ias} F(s + a)$.

(*b*) If

$$f(x) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$$

(*i*) Show that :

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \frac{\cos sx + s \sin sx}{s^2 + 1} \, ds.$$

(*ii*) Verify directly that the above integral representation converges to the value 1/2 at $x = 0$.

5. Answer the following : 20

(*a*) Using Fourier transform, solve the boundary value problem :

$$y'' - y = e^{-|x|}, \quad -\infty < x < \infty$$

$$y(x) \to 0, \; y'(x) \to 0 \text{ as } |x| \to \infty.$$

P.T.O.

(*b*) Using Fourier transform, solve the following partial differential equation under the given conditions :

$$u_{xx} = a^{-2}u_t, \qquad 0 < x < \infty,\ t > 0$$

B.C. : $u_x(0, t) = 0$, $u(x, t) \to 0$, $u_x(x, t) \to 0$ as $x \to \infty$,

I.C. : $u(x, 0) = f(x)$, $0 < x < \infty$.

6. Answer the following : 20

(*a*) Find the inverse Laplace transform of the function :

$$F(p) = \frac{p^2}{(p+2)^4}.$$

(*b*) If $\mathbf{F}_c\{f(t)\} = F_c(s)$ represents the Fourier cosine transform of $f(t)$, find :

(*i*) $\mathbf{F}_c\{e^{-at} \cos at\}$ and

(*ii*) $\mathbf{F}_c\{e^{-at} \sin at\}$.

This question paper contains **3** printed pages]

NEPHR—301—2025

FACULTY OF SCIENCE

M.Sc. (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

MATHEMATICS

(SMATES-501)

(Fluid Mechanics-I)

(Monday, 22-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time— 3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) Each question carries equal marks.

(*ii*) Figures to the right indicate full marks.

(*iii*) Question No. **1** will be compulsory.

(*iv*) Solve any *three* questions of the remaining five questions (Q. No. **2** to Q. No. **6**).

1. Attempt the following questions : 20

(*a*) Write a note on strength of vortex tube.

(*b*) Write Laplace equation in spherical polar co-ordinate system.

P.T.O.

(*c*) Write a note on line sink.

(*d*) Derive an expression for line source.

2. Attempt the following questions : 20

(*a*) Define real fluid and derive an expression for local and particle rate of change.

(*b*) Derive an equation of continuity for steady incompressible fluid.

3. Attempt the following questions : 20

(*a*) Prove that, in fluid region the pressure is same in all direction.

(*b*) Derive Euler equation of motion.

4. Attempt the following questions : 20

(*a*) Derive an expression for pressure at stagnation point of stationary sphere in a uniform stream.

(*b*) Prove that the kinetic energy of sphere moving with constant velocity in liquid which is at rest is (1/4) MV^2.

5. Attempt the following questions : 20

(*a*) State and prove Kelvin's theorem.

(*b*) Find the equation of streamline due to sources of strength m at the point A(–C, o) and B(C,o).

6. Attempt the following questions : 20

(*a*) Define steady flow. Derive an equation of continuity for steady incompressible fluid.

(*b*) Describe the irrotational motion of an incompressible liquid for which the complex potential is $w = ik \log z$.

This question paper contains **4** printed pages]

NEPHR—303—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP)

MATHEMATICS

Paper I (SMATE-501)

(Fuzzy Sets and Their Application)

(Monday, 22-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Define convex set and prove that :

$$\mu_A[\lambda_r + (1 - \lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$$

for all $r, s \in R^n$ and all $\lambda \in [0, 1]$.

P.T.O.

(*b*) Prove that :

$$\lim_{w \to \infty} i_w = \lim_{w \to \infty}\left(1 - \min\left[1, \left((1-a)^w + (1-b)^w\right)^{\frac{1}{w}}\right]\right)$$

$$= \min(a, b)$$

(*c*) Define fuzzy relation, sagittal diagram and membership matrix.

(*d*) Given :

$$Q = \begin{bmatrix} .1 & .4 & .5 & 1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix}$$

and $r = [.8, .7, .5, 0]$ find all solutions of $P \circ Q = r$ where :

$P = [P_j | j \in J]$, $Q = [q_{j,\ k} | j \in J, k \in K]$, $r = [r_k | k \in K]$.

2. Answer the following : 20

(*a*) Justify your answer, whether the law of contradiction and the law of exclusive middle are valid in fuzzy set theory.

(*b*) Consider f is a function mapping order pairs from $X_1 = \{a, b, c\}$ and $X_2 = \{x, y\}$ to $Y = \{p, q, r\}$. f specified by

$$\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} p & p \\ q & r \\ r & p \end{bmatrix} \end{array}$$

and $A_1 = .3/a + .9/b + .5/c$

$A_2 = .5/x + 1/y,$

then find $f(A_1, A_2)$.

3. Answer the following : 20

(*a*) Prove that every fuzzy complement has atmost one equilibrium.

(*b*) Show that the Sugeno complements are involved for all $\lambda \in (-1, \infty)$.

4. Answer the following : 20

(*a*) Consider the sets $X_1 = \{x, y\}$, $X_2 = \{a, b\}$ and $X_3 = \{*, \$\}$ and ternary fuzzy relation

$R(X_1, X_2, X_3) = .9/x, a, * + 0.4/x, b, * + 1/y, a, * + .7/y, a, \$ + .8/y, b, \$$

defined on $X_1 \times X_2 \times X_3$. Let

$$R_{i,\ j} = [R \downarrow \{X_i, X_j\}] \text{ and } R_i = [R \downarrow \{x_i\}]$$

for all $i, j \in \mathbf{N}$. Then find $R_{1,2}$, $R_{1,3}$, $R_{2,3}$, R_1, R_2 and R_3.

(*b*) Find the max-min composition and max-product composition for :

$$A = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}, \ B = \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix}.$$

P.T.O.

5. Answer the following : 20

(*a*) Calculate the projection $R_{1,3}$ in the following example :

(x_1, x_2, x_3)	$\mu_R(x_1, x_2, x_3)$
$(x, a, *)$	0.9
$(x, b, *)$	0.4
$(y, a, *)$	1
$(y, a, \$)$	0.7
$(y, b, \$)$	0.8

(*b*) Show that the equilibria e_{c_w} for the Yager fuzzy complements are given by the formula :

$$e_{c_w} = \left(\frac{1}{2}\right)^{\frac{1}{w}}$$

Plot this function for $w \in (0, 10]$.

6. Answer the following : 20

(*a*) Define fuzzy partial ordering. Give examples of partial ordering in the form of Hasse diagrams.

(*b*) Prove that :

$$(A \cup B) \cup C = A \cup (B \cup C)$$

for any crisp set A, B and C.

Total No. of Printed Pages:1

SUBJECT CODE NO:- NEPHR-44-2025
FACULTY OF SCIENCE & TECHNOLOGY
EXAMINATION WINTER 2025
M.Sc.(SECOND YEAR) (SEM –IV)
(COMMON PAPER)
RESEARCH PUBLICATION ETHICS (NEPPE - 1002)

[Time: 2:00 Hours] **[Max.Marks:40]**

"Please check whether you have got the right question paper."

N.B.
i) Question number 1 is compulsory.
ii) Solve any three questions from Question NO.2 to 6.

Q1 Explain: **5×2=10**
a) Nature of philosophy
b) Intellectual honesty
c) World association of medical editor's.
d) Open access publications.
e) Web of Science

Q2 a) What do you mean by philosophy? Gives the IR branches. **2x5=10**
b) Write an essay on scientific misconduct.

Q3 a) Define publication ethics? Why publication of research paper is important. Explain. **2x5=10**
b) SHERPA / ROMEO is an excellent online resource. Explain.

Q4 a) What are predatory Journals? How to identify a predatory Journals! **2×5=10**
b) What is impact Factor? How it calculate? Explain it with suitable example.

Q5 a) Give an account on violation of publications ethics. **2x5=10**
b) What is plagiarism? Describe different software of plagiarism.

Q6 Write short notes on: **4×2.5=10**
a) Scope of ethics
b) Salami slicing
c) Springer
d) h-index

This question paper contains 4 printed pages]

NEPHR—404—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP-2020 Pattern)

MATHEMATICS

Paper SMATE-551(A)

(Integral Equations)

(Tuesday, 23-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Convert the initial value problem in to an integral equation :

$$y''x + y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

P.T.O.

(*b*) Solve the integral equation :

$$y(x) = \tan x + \int_{-1}^{1} e^{\sin^{-1} x}\, y(t)\, dt.$$

(*c*) Prove that the sequence of eigenfunctions of a symmetric kernel can be made orthonormal.

(*d*) Using Laplace transform, find the resolvent kernel of the integral equation :

$$\mathrm{Y}(t) = \mathrm{F}(t) + \int_{0}^{t} e^{t-x}\, \mathrm{Y}(x)\, dx.$$

2. Answer the following : 20

(*a*) Explain the method of solving the equation :

$$y(x) = \lambda \int_{a}^{b} k(x, t)\, y(t)\, dt$$

where $k(x, t)$ is separable.

(*b*) Find the eigenvalues and the corresponding eigenfunctions of the integral equation :

$$y(x) = \lambda \int_{0}^{1} \left(2xt - 4x^2\right) y(t)\, dt.$$

3. Answer the following : 20

(*a*) Solve the Volterra integral equation of the second kind :

$$y(x) = f(x) + \lambda \int_{a}^{x} k(x, t)\, y(t)\, dt$$

by the method of successive approximations.

(*b*) With the aid of resolvent kernel, find the solution of the Volterra integral equation :

$$y(x) = e^{x^2} + \int_0^x e^{x^2 - t^2}\, y(t)\, dt.$$

4. Answer the following : 20

(*a*) Prove that the eigenfunctions of a symmetric kernel of a Fredholm integral equation of the first kind, corresponding to different eigenvalues are orthogonal.

(*b*) Prove that the multiplicity of any non-zero eigenvalue is finite for every symmetric kernel for which :

$$\int_a^b \int_a^b | k(x, t) |^2\, dx\, dt < \infty.$$

5. Answer the following : 20

(*a*) Solve the Volterra integral equation of the first and second kind with convolution type kernel using Laplace transform. Prove that if the kernel of the Volterra integral equation is of convolution type, then its resolvent kernel is also of convolution type. Using Laplace transform, obtain the expression for the resolvent kernel of Volterra integral equation of second kind with convolution type kernel.

P.T.O.

(*b*) Using Laplace transform, solve the integral equation :

$$\int_0^t \frac{Y(x)}{[t-x]^{1/3}}\, dx = t(1+t)$$

and verify your solution.

6. Answer the following : 20

(*a*) Find the characteristic values and the corresponding eigenfunctions of the homogeneous Fredholm integral equation of the second type :

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt.$$

(*b*) Using Laplace transform, solve the integral equation :

$$Y(t) = t^2 + \int_0^t Y(u) \sin(t-u)\, du$$

and verify your solution.

This question paper contains 4 printed pages]

NEPHR—404—2025

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP-2020 Pattern)

MATHEMATICS

Paper SMATE-551(A)

(Integral Equations)

(Tuesday, 23-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Convert the initial value problem in to an integral equation :

$$y''x + y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

P.T.O.

(*b*) Solve the integral equation :

$$y(x) = \tan x + \int_{-1}^{1} e^{\sin^{-1} x}\, y(t)\, dt.$$

(*c*) Prove that the sequence of eigenfunctions of a symmetric kernel can be made orthonormal.

(*d*) Using Laplace transform, find the resolvent kernel of the integral equation :

$$\mathrm{Y}(t) = \mathrm{F}(t) + \int_{0}^{t} e^{t-x}\, \mathrm{Y}(x)\, dx.$$

2. Answer the following : 20

(*a*) Explain the method of solving the equation :

$$y(x) = \lambda \int_{a}^{b} k(x, t)\, y(t)\, dt$$

where $k(x, t)$ is separable.

(*b*) Find the eigenvalues and the corresponding eigenfunctions of the integral equation :

$$y(x) = \lambda \int_{0}^{1} \left(2xt - 4x^2\right) y(t)\, dt.$$

3. Answer the following : 20

(*a*) Solve the Volterra integral equation of the second kind :

$$y(x) = f(x) + \lambda \int_{a}^{x} k(x, t)\, y(t)\, dt$$

by the method of successive approximations.

(*b*) With the aid of resolvent kernel, find the solution of the Volterra integral equation :

$$y(x) = e^{x^2} + \int_0^x e^{x^2 - t^2}\, y(t)\, dt.$$

4. Answer the following : 20

(*a*) Prove that the eigenfunctions of a symmetric kernel of a Fredholm integral equation of the first kind, corresponding to different eigenvalues are orthogonal.

(*b*) Prove that the multiplicity of any non-zero eigenvalue is finite for every symmetric kernel for which :

$$\int_a^b \int_a^b |k(x, t)|^2\, dx\, dt < \infty.$$

5. Answer the following : 20

(*a*) Solve the Volterra integral equation of the first and second kind with convolution type kernel using Laplace transform. Prove that if the kernel of the Volterra integral equation is of convolution type, then its resolvent kernel is also of convolution type. Using Laplace transform, obtain the expression for the resolvent kernel of Volterra integral equation of second kind with convolution type kernel.

P.T.O.

(*b*) Using Laplace transform, solve the integral equation :

$$\int_0^t \frac{Y(x)}{[t-x]^{1/3}}\, dx = t(1+t)$$

and verify your solution.

6. Answer the following : 20

(*a*) Find the characteristic values and the corresponding eigenfunctions of the homogeneous Fredholm integral equation of the second type :

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\, dt.$$

(*b*) Using Laplace transform, solve the integral equation :

$$Y(t) = t^2 + \int_0^t Y(u) \sin(t-u)\, du$$

and verify your solution.

This question paper contains **3** printed pages]

NEPHR—407—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

MATHEMATICS

Paper II (SMATE-551)

(Fuzzy Sets and Their Applications)

(Tuesday, 23-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that every possibility measure π on $\mathbf{P}(X)$ can be uniquely determined by a possibility distribution function :

$$r : X \to [0, 1]$$

via the formula :

$$\pi(A) = \max_{x \in A} r(x)$$

for each $A \in \mathbf{P}(X)$.

P.T.O.

(*b*) Define unit of fuzziness and derive formula for unit of fuzziness.

(*c*) Calculate U(r) for the possibility distribution :

$$r = (1, 1, 0.8, 0.7, 0.7, 0.7, 0.4, 0.3, 0.2, 0.2).$$

(*d*) Describe rule for A fuzzy logic controller (FLC) in employed.

2. Answer the following : 20

(*a*) A belief measure is a function

$$\text{Bel} : \mathbf{P}(X) \to [0, 1],$$

then prove that :

$$\text{Bel}(A_1 \cup A_2 \cup \ldots\ldots \cup A_n) \geq \sum_i \text{Bel}(A_i) - \sum_{i<j} \text{Bel}(A_i \cap A_j) + \ldots\ldots + (-1)^{n+1} \text{Bel}(A_1 \cap A_2 \cap \ldots\ldots \cap A_n).$$

(*b*) Explain combination of consonant bodies of evidence by the minimum operator versus Dempster's rule.

3. Answer the following : 20

(*a*) Prove that :

$$\lim_{i \to \infty} \frac{q(i)}{i} = I(N);$$

where $I(N) = \log_2 N$ for $N > 2$, $q(i)$ is an integer.

(*b*) Let m_X and m_Y be marginal basic assignment on set X and Y respectively and let m be a joint basic assignment on X × Y such that $m(A \times B) = m_X(A) \cdot m_Y(B)$ for all $A \in \mathbf{P}(X)$ and $B \in \mathbf{P}(Y)$. Then prove that $E(m) = E(m_X) + E(m_Y)$.

4. Answer the following : 20

(*a*) Write a note on 'General Simplification problem with illustration'.

(*b*) Derive the formula for maximum entropy principle.

5. Answer the following : 20

(*a*) Define normalization for the degree of non-specificity of the original basic assignment.

(*b*) Draw figure of An overview of uncertainty measure.

6. Answer the following : 20

(*a*) Explain the fuzzy model of decision-making proposed by Bellman and Zadeh with simple example.

(*b*) Define Pseudo-frequencies and derive crisp possibility distribution in which :

$$r(s) = \begin{cases} 1, & \text{when } N(s) = \max_{x \in s} N(X) \\ 0, & \text{otherwise} \end{cases}$$

This question paper contains 4 printed pages]

NEPHR—129—2025

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2025

(NEP 2020)

MATHEMATICS

SMATC-551

(Numerical Analysis)

(Tuesday, 16-12-2025) **Time : 2.00 p.m. to 5.00 p.m.**

Time— 3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

(*v*) Scientific calculator is allowed.

1. Attempt the following : 20

(*a*) Perform three iterations of the Newton-Raphson method to obtain the approximate value of $17^{\frac{1}{3}}$. Take the initial approximation as $x_0 = 2$.

P.T.O.

(b) Obtain the inverse of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

by using the partition method.

(c) Determine the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

by using an iterative method. Give the approximate inverse is :

$$B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}.$$

(d) Obtain the approximate value of sin (0.15) by using Lagrange interpolation. Use the data, sin(0.1) = 0.09983 and sin (0.2) = 0.19867.

2. Attempt the following : 20

(a) Discuss in detail the Chebyshev method to obtain the root of the equation $f(x) = 0$.

(b) Obtain the smallest positive root of the equation :

$$f(x) = x^3 - 5x + 1 = 0$$

by the Newton-Raphson method. Perform four iterations.

3. Attempt the following : 20

(*a*) Explain in detail the Gauss elimination method of solving the system of equations Ax = b.

(*b*) Solve the system of equations :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

by using the Cholesky method.

4. Attempt the following : 20

(*a*) Discuss in detail the Gauss-Seidel method of solving the system of equations Ax = b. Also obtain its error format.

(*b*) Solve the system of equations :

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_2 = -4$$

by using Jacobi iteration method. Take initial approximation as $x^{(0)} = [0.5 -0.5 -0.5]^T$. Perform three iterations.

5. Answer the following : 20

(*a*) Discuss in detail Lagrange's quadratic interpolation.

P.T.O.

(*b*) The function $f(x) = \sin x$ is defined on the interval [1, 3] :

(*i*) Obtain the Lagrange linear interpolating polynomial in the interval [1, 3] and find the bound on the truncation error. Obtain the approximate value of $f(1.5)$ and $f(2.5)$.

(*ii*) Divide the interval [1, 3] into two subintervals [1, 2] and [2, 3]. Obtain the Lagrange linear interpolating polynomial in each subinterval and find the bound on the truncation error. Find the approximate value of $f(1.5)$ and $f(2.5)$.

6. Answer the following : 20

(*a*) Discuss in detail the partition method to obtain the inverse of the matrix. Obtain the inverse of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

by using the partition method.

(*b*) Discuss in detail the Gauss-Seidel method of solving the system of equations $Ax = b$. Also obtain its error format.