This question paper contains **3** printed pages]

# NEPSST—1—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.Sc. (First Year) (First Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP 2020)

RESEARCH METHODOLOGY

Paper NEPRN-1001

**(Wednesday, 16-4-2025)** **Time : 10.00 a.m. to 12.30 p.m.**

*Time*—2½ *Hours* *Maximum Marks*—60

*N.B.* :— (*i*) Question No. **1** is compulsory.

(*ii*) Of the remaining solve any *three* questions.

(*iii*) Calculator and log table is allowed.

1. Attempt any *three* of the following : 15

(*a*) Qualities of good research.

(*b*) Features of good design.

(*c*) ANOVA

(*d*) Types of data.

2. (*a*) What is research ? Explain steps involved in research process. 8

(*b*) Discuss interview as a technique of data collection. 7

P.T.O.

3. (*a*) Calculate the Mean, Median and Mode of the following data : 8

| **Class Interval (CI)** | **Frequency (F)** |
|---|---|
| 50–54 | 2 |
| 45–49 | 5 |
| 40–44 | 8 |
| 35–39 | 7 |
| 30–34 | 10 |
| 25–29 | 5 |
| 20–24 | 9 |
| 15–19 | 2 |
| 10–14 | 1 |
| 5–9 | 1 |

(*b*) What is hypothesis ? Give the characteristics of good research hypothesis. 7

4. (*a*) Describe non-probability and probability sampling. 8

(*b*) Calculate the Chi-square value of the following data : 7

| Excellent | Average | Poor | Total |
|---|---|---|---|
| 58 | 32 | 30 | 120 |

5. (*a*) Define case study. Give their components. 8

(*b*) Explain extraneous variable. 7

6. Write short notes on : 15

(*a*) Descriptive types of research

(*b*) Non-parametric test

(*c*) Primary data sources.

This question paper contains **3** printed pages]

# NEPSST—50—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (First Year) (First Semester) EXAMINATION

## APRIL/MAY, 2025

APPLIED MATHEMATICS

(SAMATC-401)

(Modern Algebra)

**(Saturday, 19-04-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time—Three Hours* *Maximum Marks*—80

*Note :—* (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 4×5=20

(*a*) If the order of a finite group G is divisible by a prime number P, then G has an element of order P.

(*b*) Find the non-isomorphic abelian group of order $360 = 2^3 \cdot 3^2 - 5^1$

(*c*) Show that a finite integral domain is a division ring

(*d*) Write down a composition series for the klein four group.

P.T.O.

2. Answer the following : 20

(*a*) If $\phi : G \to G'$ be a homomorphism of groups, then :

$$G / \ker \phi \simeq \text{Im } \phi .$$

(*b*) If H and K be subgroups of a group (G, *), then HK is a group of G if and only if HK = KH.

3. Answer the following : 20

(*a*) Prove that every group of order $P^2$(P prime) is abelian.

(*b*) Prove that every finite group has a composition series.

4. Answer the following : 20

(*a*) If A be a finite abelian group and P be a prime, P divides $|A|$, then A has an element of order P.

(*b*) If G be a finite group and P be a prime and $P^m$ divides $|G|$, then G has a subgroup of order $P^m$.

5. Answer the following : 20

(*a*) If $f : R \to S$ be a homomorphism of a ring R into a ring S, then $\ker f = (0)$ if and only if $f$ is $1 - 1$.

(*b*) Prove that every Eucledian domain is PID.

6. Answer the following : 20

(*a*) If R is commutative ring with unity and M is ideal of R, then prove that M is maximal ideal if and only if R/M is field.

(*b*) Prove that an non-empty subset of S of a ring R is a subring if and only if for all $a, b \in S$ and $a - b \in S, a, b \in S$.

This question paper contains **4** printed pages]

# NEPSST—176—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

**M.A./M.Sc. (First Year) (First Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(CBCS/New Pattern)**

MATHEMATICS

SMATC-402

(Real Analysis)

**(Tuesday, 22-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

***N.B.*** :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. N. **2** to Q. N. **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt the following : 20

(*a*) If $f \in R(a)$ and $g \in R(a)$, on $[a, b]$, then show that $fg \in \mathbf{R}(a)$.

(*b*) If $\{f_n\}$ and $\{g_n\}$ converges uniformly on a set E, then prove that $\{f_n + g_n\}$ converges uniformly on E.

(*c*) Does the every member of an equicontinuous family is uniformly continuous ? Justify your answer.

P.T.O.

(*d*) Show that the function $f(x, y) = |xy|$ is differentiable at $\bar{0}$ but is not of class $C^1$ in any neighborhood of $\bar{0}$.

2. Answer the following : 20

(*a*) If $\gamma'$ is continuous on $[a, b]$, then prove that $\gamma$ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

(*b*) If $f(x) = 0$ for all irrational $x$, $f(x) = 1$ for all rational $x$, show that $f \notin \mathbf{R}(\alpha)$ on $[a, b]$ for any $a < b$.

3. Answer the following : 20

(*a*) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let $x$ be a limit point of E and suppose that :

$$\lim_{t \to x} f_n(t) = A_n \quad n = 1, 2, 3, \ldots\ldots\ldots$$

Then show that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{t \to x} A_n$.

(*b*) (*i*) For $m = 1, 2, 3, \ldots\ldots\ldots$ , $n = 1, 2, \ldots\ldots\ldots\ldots$ , let $S_{mn} = \dfrac{m}{m+n}$

Show that :

$\lim_{n \to \infty} \lim_{m \to \infty} s_{mn} \neq \lim_{m \to \infty} \lim_{n \to \infty} S_{mn}$.

(*ii*) Let $f_n(x) = \dfrac{\sin x}{\sqrt{n}}$, $(x \in R, n = 1, 2, \ldots\ldots)$, then prove that $\lim_{n \to \infty} f_n'(x) \neq f'(x)$.

4. Answer the following : 20

(*a*) Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converge in the segement S = (– R, R). Let E be the set of all $x \in S$ at which.

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n.$$

If E has a limit point in S, then prove that $a_n = b_n$ for $n = 0, 1, 2$ .... hence holds for all $x \in S$.

(*b*) If $f$ is continuous on [0, 1] and if $\int_0^1 f(x) x^n dx = 0$, $n = 0, 1, 2, \ldots\ldots$, show that $f(x) = 0$ on [0, 1].

5. Answer the following : 20

(*a*) (*i*) Let A be open in $R^n$, let $f : A \to R^n$, let $f(\bar{b}) = \bar{a}$. Suppose that $g$ maps a neighborhood of $\bar{b}$ into $R^n$ and $g(\bar{b}) = \bar{a}$ and $g(f(\bar{x})) = \bar{x}$ for all $\bar{x}$ in a neighborhood of $\bar{a}$. If $f$ is differentiable at $\bar{a}$ and if $g$ is differentiable at $\bar{b}$, then prove that :

$$Dg(\bar{b}) = [Df(a)]^{-1}.$$

(*ii*) Let $A \subset R^m$, let $f : A \to R$. If $f$ is differentiable at $\bar{a}$, then show that :

$$Df(\bar{a}) = [D_1 f(\bar{a}), D_2 f(\bar{a}), \ldots\ldots\ldots, D_m f(\bar{a})].$$

(*b*) Let $f : R^2 \to R^2$ be defined by the equation

$f(x, y) = (e^x \cos y, e^x \sin y)$

(*i*) Show that $f$ is one to one on the set A consisting of all $(x, y)$ with $0 < y < 2\pi$.

(*ii*) What is the set B = F(A) ?

If $g$ is the inverse function, find $D_g(0, 1)$.

6. Answer the following : 20

(*a*) State the prove fundamental theorem of Calculus.

Also, let $f_n(x) = n^2 x(1 - x^2)^n$, $0 \le x \le 1$ and $n = 1, 2, 3, .....$, show that

$$\lim_{n \to 0} \left[ \int_0^1 f_n(x) dx \right] \neq \int_0^1 \left[ \lim_{n \to \infty} f_n(x) \right] dx$$

(*b*) State the Stone-Weierstrass theorem.

Define $f : R^2 \to R$ by setting $f(\overline{0}) = \overline{0}$ and $f(x, y) = \dfrac{x^2}{x^4 + y^{2'}}$ if $f(x, y) \neq \overline{0}$.

Show that all directional derivative of $f$ exist to $\overline{0}$ but that $f$ is not differentiable at $\overline{0}$.

This question paper contains **3** printed pages]

# NEPSST—294—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (First Year) (First Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020 Pattern)

MATHEMATICS

Paper SMATC-403

(Complex Analysis)

**(Thursday, 24-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find all the values of $i^{1/2}$ and $5^{1/2}$.

(*b*) Find the length of the curve C : $z(t) = e^{2it}$ $\left(0 \le t \le \frac{\pi}{2}\right)$.

(*c*) $\int_C \frac{e^z}{(z-2)^2}\,dz$ where, C : $|z| = 3$.

P.T.O.

(*d*) Find the principal part of the Laurent's expansion for the function $f(z) = \frac{z}{z^2 + 4}$ valid in the neighbourhood of $z = -2i$.

2. Answer the following : 20

(*a*) Prove that, for given three distinct points $z_1$, $z_2$ and $z_3$ in extended $z$-plane and three distinct points $w_1$, $w_2$ and $w_3$ in extended $w$-plane there exist a unique bilinear transformation $w = T(z)$ such that $T(z_k) = w_k$ for $k = 1, 2, 3$.

(*b*) Find all the values of :

(*i*) $\log(1 - i)$

(*ii*) $\sin^{-1}(0)$

(*iii*) $\cos^{-1}\frac{\sqrt{2}}{2}$.

3. Answer the following : 20

(*a*) Define analytic function. Prove that the necessary condition for the differentiability of the function $f(z)$ at a point $z = a$ is that $f_{\bar{z}} = 0$.

(*b*) Calculate $\int_C z^2\, dz$ along the curves :

(*i*) $C : z_1(t) = t + it \ (0 \le t \le 1)$

(*ii*) $C : z_2(t) = t + it^2 \ (0 \le t \le 1)$.

4. Answer the following : 20

(*a*) Define entire function. State and prove Cauchy's inequality.

(*b*) Evaluate :

(*i*) $\int_C \frac{3z^4 + 2z - 6}{(z-2)^3}\, dz$ where, C : $|z| = 3$.

(*ii*) $\int_C \frac{z - 3\cos z}{(z - \pi/2)^2}\, dz$ where, C : $|z| = 2$.

5. Answer the following : 20

(*a*) State and prove Residue Theorem.

(*b*) Prove that :

$$\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}.$$

6. Answer the following : 20

(*a*) State and prove Liouville's theorem.

(*b*) Find [Res : $f(z)$; $z = 1$] for the function $f(z) = \frac{z^4 - z^3 + 17z + 12}{(z-1)^3}$. Also evaluate $\int_C f(z)dz$.

This question paper contains **3** printed pages]

# NEPSST—80—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.Sc. (First Year) (First Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020)

MATHEMATICS

Paper–SMATC-401

(Algebra)

**(Saturday, 19-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) Question No. **1** is compulsory.

(*ii*) Solve any *three* of the remaining five questions Q. No. **2** to Q. No. **6**.

1. (*a*) Let G and H be groups with identities $e$ and $e'$ respectively, and let $\phi : G \to H$ be a homomorphism. Then : 20

(*i*) $\phi(e) = e^1$

(*ii*) $\phi(x^{-1}) = [\phi(x)]^{-1}$ for each $x \in G$.

P.T.O.

(*b*) An abelian group G has a composition series if and only if G is finite.

(*c*) If each element $\neq e$ of a finite group G is of order 2, then $|G| = 2^n$ and $G \simeq C_1 \times C_2 \times ..... \times C_n$, where C*i* are cyclic and $|C_i| = 2$.

(*d*) Prove that, the centre of a ring is a subring.

2. (*a*) Let G be a group. A nonempty subset H of G is a subgroup of if and only if either of the following holds : 2×10

(*i*) For all $a, b \in H$, $ab \in H$ and $a^{-1} \in H$

(*ii*) For all $a, b \in H$, $ab^{-1} \in H$.

(*b*) Let G be a finite group. Then the order of any subgroup of G divides the order of G.

3. (*a*) Let G be a group. Then the following are true : 2×10

(*i*) The set of conjugate classes of G is a partition of G.

(*ii*) $|C(a)| = [G : N(a)]$.

(*b*) Let G be a finite group acting on a finite set X. Then the number K of orbits in X under G is :

$$K = \frac{1}{|G|} \sum_{g \in G} |X_g|.$$

4. (*a*) Let A be a finite abelian group. Then there exists a unique list of integers $m_1, m_2, ....., m_k$ (all > 1) such that $|A| = m_1 ..... m_k$, $m_2 | m_2 | ..... | m_k$, and A = $C_1 \oplus ..... \oplus C_k$, where $C_1, ....., C_k$ are cyclic subgroups of A of order $m_2, ...., m_k$ respectively. Consequently $A \simeq Z_{m_1} \oplus ..... \oplus Z_{m_k}$. 2×10

(*b*) Let G be a finite group, and let $p$ be a prime. If $p^m$ divides $|G|$, then G has a subgroup of order $p^m$.

5. (*a*) If a ring R has unity, then every ideal I in the matrix ring $R_n$ is of the form $A_n$, where A is some ideal in R. 2×10

(*b*) Let $(A_i)_{i\in A}$ be a family of right ideals in a ring R. Then $\bigcap_{i\in A} A_i$ is also a right ideal.

6. (*a*) Every group is isomorphic to a permutation group. 2×10

(*b*) An irreducible element in a commutative principal ideal domain (PID) is always prime.

This question paper contains **3** printed pages]

# NEPSST—480—2025

## FACULTY OF SCIENCE

**M.Sc. (First Year) (First Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP 2020 Pattern)**

MATHEMATICS

Paper SMATE-401 (A)

(Ordinary Differential Equations)

**(Saturday, 26-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. 2 to Q. No. 6.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Solve $y'' + \omega^2 y = 0$.

(*b*) Solve $x^2y'' + 2xy' - 6y = 0$.

(*c*) Solve $y^{(3)} - 4y' = 0$.

(*d*) Verify that $\phi_1(x) = x(x > 0)$ satisfies the equation $x^2y'' - xy' + y = 0$ and find the second independent solution.

P.T.O.

2. Answer the following : 20

(*a*) Prove that every solution $\psi$ of $L(y) = b(x)$ on I is $\psi = \psi_p + c_1\phi_1 + c_2\phi_2$, where $\psi_p$ is particular solution, where $\phi_1, \phi_2$ are two linearly independent solutions of $L(y) = 0$ and $c_1$, $c_2$ are constants.

(*b*) Prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$, where $\phi_1, \phi_2$ are two linearly independent solutions of $L(y) = 0$ on an interval I containing a point $x_0$.

3. Answer the following : 20

(*a*) Derive two solutions of Legendre equation

(*b*) $\phi_1, \phi_2 \ldots \phi_n$ be $n$ solutions of $L(y) = 0$ on an interval I, and $x_0$ be any point in I, then prove that :

$$W(\phi_1, \phi_2 \ldots \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)dt\right] W(\phi_1, \phi_2 \ldots \phi_n)(x_0)$$

4. Answer the following : 20

(*a*) Find *two* solutions of Bessel equation.

(*b*) Find *two* solutions of Euler's equation.

5. Answer the following : 20

(*a*) Prove that $M(x, y) + N(x, y)y' = 0$ is exact on some rectangle R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(*b*) Prove that a function $\phi$ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I.

6. Answer the following : 20

(*a*) Solve $y^{(3)} - 4y' = 0$.

(*b*) Solve $y''' - y' = x$.

This question paper contains **4** printed pages]

# NEPSST—481—2025

**FACULTY OF ARTS/SCIENCE**

**M.A./M.Sc. (First Year) (First Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP 2020 Pattern)**

MATHEMATICS

SMATE-401-B

(Discrete Mathematics)

**(Saturday, 26-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

***N.B.*** *:—* (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

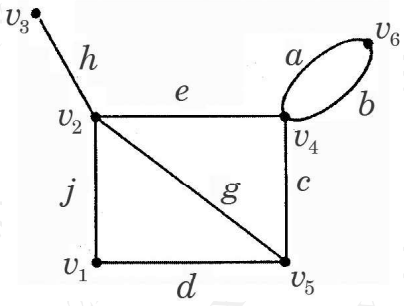
(*a*) Let $a$ and $b$ be two elements in a lattice (A, $\leq$). Show that $a \wedge b = b$ if and only if $a \vee b = a$.

(*b*) Write a short note on isomorphism of graphs.

(*c*) Prove that every connected graph has at least one spanning tree.

P.T.O.

(*d*) Obtain an incidence matrix of the following graph :




2. Answer the following : 20

(*a*) For any $a$, $b$, $c$, $d$ in a lattice (A, $\leq$) if $a \leq b$, and $c \leq d$, then prove that $a \vee c \leq b \vee d$ and $a \wedge \leq b \wedge d$.

(*b*) Define the universal upper and lower bounds in a lattice with a suitable example. Let (A, $\leq$) be a lattice with universal upper and lower bounds 1 and 0. Then for every $a$ in (A, $\leq$), prove that :

$$a \vee 1 = 1 \quad a \wedge 1 = a$$

$$a \vee 0 = a \quad a \wedge 0 = 0$$

3. Answer the following : 20

(*a*) Define a connected and a disconnected graph with a suitable example. Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets $V_1$ and $V_2$ such that there exists no edge in G whose one end vertex is in the subset $V_1$ and the other in subset $V_2$.

(*b*) Define a unicursal graph with suitable examples. In a connected graph G with exactly $2k$ odd vertices, prove that there exists $k$ edge-disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

4. Answer the following : 20

(*a*) Prove that there exists one and only one path between every pair of vertices in a tree, T. Conversely, in a graph G, if there is one and only one path between every pair of vertices, then prove that G is a tree.

(*b*) Define the distance between two vertices and eccentricity of a vertex in a connected graph. Prove that every tree has either one or two centres.

5. Answer the following : 20

(*a*) Let A(G) be an incidence matrix of a connected graph G with $n$ vertices. Then prove that an $(n-1) \times (n-1)$ submatrix of A(G) is non-singular if and only if the $n-1$ edges corresponding to the $n-1$ columns of this matrix constitute a spanning tree in G.

(*b*) Define a directed graph with suitable example. Define in-degree and out-degree of a vertex in a diagraph. Verify with a suitable example that in any diagraph, the sum of all in-digrees is equal to the sum of all out-degrees.

P.T.O.

6. Answer the following : 20

(*a*) Let (A, $\vee$, $\wedge$, $^{-}$) be a finite Boolean algebra. Let $b$ be any non-zero element of A and let $a_1$, $a_2$, ....., $a_k$ be all the atoms of A such that $a_i \leq b$. Then prove that $b = a_1 \vee a_2 \vee \ldots\ldots \vee a_k$ is the unique way to represent $b$ as a join of atoms.

(*b*) Define vertex and edge connectivity of a connected graph with suitable examples. Prove that the vertex connectivity of a graph G cannot exceed the edge connectivity of G.

This question paper contains **2** printed pages]

# NEPSST—482—2025

## FACULTY OF SCIENCE

### M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

### APRIL/MAY, 2025

MATHEMATICS

(Dynamics and Continuum Mechanics–I)

**(Saturday, 26-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) Figures to the right indicate full marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

1. Answer the following : 20

(*a*) Explain scalar moment about an axis.

(*b*) Show that the earth's gravitational field is conservative.

(*c*) The force having components (–1, 2, 1) acts to the point (1, 1, –2), then find the vector moment about the point having co-ordinate (0, 1, –1) and physical moment about axis to the point having direction cosine $(1/3, -2/3, 2/3)$.

(*d*) Explain co-planar distribution.

P.T.O.

2. Answer the following :

(*a*) Show that the centroid of the system is unique. 10

(*b*) Find the radial and transverse component of velocity and acceleration of particle moving in a plane. 10

3. (*a*) Find the vector angular velocity of a rigid body which is rotating about origin at any point P. 10

(*b*) Find the vector velocity and vector acceleration of particle with moving axes. 10

4. (*a*) Find the angular momentum of rigid body using the centroid of the system. 10

(*b*) Determine the moment of inertia of the distribution about the axis through O having direction cosines $[\lambda, \mu, \gamma]$ in terms of these d.c.s. and A, B, C, D, E, F. 10

5. (*a*) Derive an expression for kinetic energy of the rigid body with respect to the principal axes. 10

(*b*) A uniform rod AB moves so that A and B have velocities $\bar{V}_A$ and $\bar{V}_B$ at any instant. Show that kinetic energy is : 10

$$T = \frac{M}{6}\left[\bar{V}_A^2 + \bar{V}_B^2 + \bar{V}_A \cdot \bar{V}_B\right]$$

6. (*a*) Find the vector moment of couple due the force. 10

(*b*) A uniform rectangular lamina ABCD such that AB = 2*a* and BC = 2*b*. Find the direction of principle. 10

This question paper contains **3** printed pages]

# NEPSST—483—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.Sc. (First Year) (First Semester) EXAMINATION

## APRIL/MAY, 2025

MATHEMATICS

SMATE-401(D)

(Theory of Probability)

**(Saturday, 26-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

***N.B.*** :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of table for area under standard normal curve is allowed.

1. Answer the following : 20

(*a*) If $E_1, E_2, E_n, \ldots\ldots, En$ are mutually disjoint events with $P(E_i) \neq 0$, ($i = 1, 2, \ldots\ldots n$), then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} E_i$, such that $P(A) > 0$, then prove that :

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)} = \frac{P(E_i)P(A/E_i)}{P(A)}$$

P.T.O.

(*b*) Let the random variable X have the following distribution :

$P(X = 0) = P(x = 2) = p$ ; $P(x = 1) = 1 - 2p$ for $0 \leq p \leq 1/2$. For what value of $p$ is the variance of X a maximum ?

(*c*) In a Poisson frequency distribution, frequency corresponding to 3 successes is 2/3 times frequency corresponding to 4 successes. Find the mean and standard deviation of the distribution.

(*d*) Define normal distribution. Find its cumulant generating function.

2. Answer the following : 20

(*a*) Show that the mathematical expectation of the product of a number of independent random variables is equal to the product of their expectations. 20

(*b*) A box contains '*a*' white and '*b*' black balls. '*c*' balls are drawn at random. Find the expected value of the number of white balls drawn.

3. Answer the following : 20

(*a*) Show that the recurrence relation for the moments of Bionomial distribution.

(*b*) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain :

(*i*) no defective, and

(*ii*) at least two defectives.

[Given : $e^{-0.5} = 0.6065$]

4. Answer the following : 20

(*a*) Explain in detail the area property of the normal probability curve.

(*b*) Define normal distribution. Find its moment generating function.

5. Answer the following :

(*a*) For $n$ events $A_1$, $A_2$, ..... $A_n$, prove that

$$P\left(\bigcup_{i=1}^{n} Ai\right) \geq \sum_{i=1}^{n} P(Ai) - \sum_{1 \leq i \leq j \leq n} P(Ai \cap Aj)$$

(*b*) A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner :

(*i*) There must be one from each category.

(*ii*) It should have at least one from the purchase department.

(*iii*) The chartered accountant must be in the committee.

6. (*a*) Answer the following : 20

Prove that sum of independent of Poisson variate is also a Poisson variate.

(*b*) X is a normal variate with mean 30 and S.D. 5. Find the probabilities that :

(*i*) $26 \leq x \leq 40$

(*ii*) $x \geq 45$

(*iii*) $|x - 30| > 5$.

This question paper contains **4** printed pages]

# NEPSST—23—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP 2020)

APPLIED MATHEMATICS

SAMATC-451

(Linear Algebra)

**(Thursday, 17-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

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*Time*—3 *Hours* *Maximum Marks*—80

***N.B.*** :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that the set $W = \{(a_1, a_2, a_3) \in R^3 : a_1 = 3a_2 \text{ and } a_3 = -a_2\}$ is subspace of $R^3$.

(*b*) Prove that the transformation :

$T : R^3 \to R^2$ defined by

$T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$ is linear transformation.

P.T.O.

(*c*) Find the rank of the following matrix :

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(*d*) If $<v_1, v_2, \ldots\ldots\ldots, v_k\}$ be an orthogonal set in $v$, and let $a_1, a_2, \ldots\ldots, a_k$ be scalars, then prove that :

$$\left\| \sum_{i=1}^{k} a_i v_i \right\|^2 = \sum_{i=1}^{k} (a_i)^2 \left\| v_i \right\|^2 .$$

2. Answer the following : 20

(*a*) Prove that, the span of any subset S of a vector space $v$ in a subspace of $v$. Moreover any subspace of $v$ that contains '$s$' must also contain the span of $s$.

(*b*) Let V be a vector space and $S_1 \subseteq S_2 \subseteq V$. If $S_1$ is linearly linearly dependent, then prove that $S_2$ is linearly dependent.

3. Answer the following : 20

(*a*) Let V and W be vector spaces, and T : V $\rightarrow$ W be linear. If V is finite dimensional, then prove that nullity (T) + rank (T) = dim (V).

(*b*) If V and W are vector spaces and T : V $\rightarrow$ W is linear, then prove that N(T) and R(T) are subspace of V and W, respectively.

4. Answer the following : 20

(*a*) Prove that elementary matrices are invertible, and the inverse of an elementary matrix is an elementary matrix of the same type.

(*b*) If T : V $\rightarrow$ W is a linear transformation between finite dimensional vector space and let $\beta$ and $\gamma$ be ordered bases for V and W, respectively, then prove that rank(T) = rank$\left([T]_{\beta}^{\gamma}\right)$.

5. Answer the following : 20

(*a*) Let V be an inner product space and S = $<v_1, v_2, \ldots\ldots, v_k\}$ be an orthogonal subset of V consisting of nonzero vectors that If $\gamma \in$ span(S), then prove

$$y = \sum_{i=1}^{n} \frac{< y_1 v_i >}{\|v_i\|^2} v_i .$$

(*b*) If V is an inner product space and S = $<w_1, w_2, \ldots\ldots, w_n\}$ is a linearly independent subset of V. Define S$'$ = $\{v_1, v_2, \ldots\ldots, v_n\}$, where $v_1 = w_1$ and

$$v_k = w_k - \sum_{j=1}^{k-1} \frac{< w_k v_i >}{\|v_j\|^2} v_j \text{ For } 2 \leq k \leq n.$$

Then prove that S$'$ is an orthogonal set of non-zero vectors such that span(S$'$) = Span(S).

6. Answer the following : 20

(*a*) Prove that a set S of vectors linearly independent iff each finite subset of S is linearly independent.

(*b*) If T : V $\rightarrow$ W and U : W $\rightarrow$ Z are linear transformations on finite dimensional vector spaces V, W and Z, and A and B are matrices such that the product AB is defined, then prove that :

(*a*) rank(UT) $\leq$ rank (U)

(*b*) rank(UT) $\leq$ rank (T).

This question paper contains **3** printed pages]

# NEPSST—128—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020)

MATHEMATICS

Paper–SMATC-452

(Measure and Integration Theory)

**(Monday, 21-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Show that if $F \in M$ and $m^*(F \Delta G) = 0$, then G is measurable.

(*b*) Show that if $f, g \in BV$ $[a, b]$, then $f.g \in BV$ $[a, b]$.

(*c*) Write a short note on Hereditary class.

(*d*) Show that $v^+ = \frac{1}{2}(v + |v|), v^- = \frac{1}{2}(|v| - v)$, provided $v$ is finite valued.

P.T.O.

2. Answer the following : 20

(*a*) State and prove Lebesgue's monotone convergence theorem.

(*b*) Show that every countable set has measure zero. Also show that if $f$ and $g$ are measurable, $|f| \leq |g|$ a.e., and $g$ is integrable, then $f$ is integrable.

3. Answer the following : 20

(*a*) Show that, a function $f \in \mathrm{BV}\ [a, b]$ if and only if $f$ is the difference of two finite valued monotone increasing function on $[a, b]$, where $a$ and $b$ are finite.

(*b*) Let $f$ be defined by $f(x) = x \sin(1/x)$ for $x \neq 0$, $f(0) = 0$, find the four derivatives at $x = 0$.

4. Answer the following : 20

(*a*) Let $\mu^*$ be an outer measure on H(R) and let $S^*$ denote the class of $\mu^*$ measurable sets. Then prove that $S^*$ is a $\sigma$-ring and $\mu^*$ restricted to $S^*$ is a complete measure.

(*b*) Show that, $\mathrm{H(R)} = \left\{ \mathrm{E} \,!\, \mathrm{E} \subseteq \bigcup_{n=1}^{\infty} \mathrm{E}_n, \mathrm{E}_n \in \mathrm{R} \right\}$.

5. Answer the following : 20

(*a*) State and prove the Jordan decomposition theorem.

(*b*) Show that the following conditions on the measures $\mu$ and $v$ on $\lceil [\mathrm{X}, \mathrm{S}] \rceil$ are equivalent :

(*i*) $v << \mu$

(*ii*) $|v| << |\mu|$

(*iii*) $v^+ << \mu$ and $v^- << \mu$.

6. Answer the following : 20

(*a*) Show that, if $m^*(A) = 0$, then prove that $m^*(A \cup B) = m^*(B)$, for any set B. Also, show that, if $f \in BV[a, b]$, then $f$ is bounded on $[a, b]$.

(*b*) (*i*) Let $f = g$ *a.e* $(\mu)$, where $\mu$ is a complete measure. Show that if $f$ is measurable, so is $g$.

(*ii*) Show that, a countable union of sets positive with respect to a signed measure $v$ is positive set.

This question paper contains **2** printed pages]

# NEPSST—225—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

**M.A./M.Sc. (First Year) (Second Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP 2020)**

MATHEMATICS

Paper SMATC-453

(Topology)

**(Wednesday, 23-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If $\mathbf{T_1}$ and $\mathbf{T_2}$ are topologies on X, show that $\mathbf{J_1} \cap \mathbf{J_2}$ is topology on X.

(*b*) If A is subspace of X, then show that the inclusion mapping $J : A \to X$ is continuous function.

(*c*) If X is finite set, then show that a discrete topological space is compact.

(*d*) Show that the set of Real numbers with usual topology is normal space.

P.T.O.

2. Answer the following : 20

(*a*) If Y is the subspace of (X, **T**) and $\overline{A}$ be the closure of A in X, then prove that, closure of A in Y is $\overline{A} \cap Y$.

(*b*) If **B** is a basis for a topology of X and **C** is a basis for the topology for *y*, then the collection **D** = $\{B \times C \mid B \in \mathbf{B}$ and $C \in \mathbf{C}\}$ is a basis for the topology of X × Y.

3. Answer the following : 20

(*a*) If X and Y are topological spaces, then prove that the following conditions are equivalent :

(*i*) $f : X \to Y$ is continuous.

(*ii*) If V is closed in Y, then $f^{-1}(V)$ is closed in X.

(*b*) Prove that the product of connected set is a connected set.

4. Answer the following : 20

(*a*) Prove that closed subset of compact topological space (X, **T**) is compact set.

(*b*) Prove that every compact topological space (X, **T**) is limit point compact.

5. Answer the following : 20

(*a*) Prove that subspace of Regular space is regular space.

(*b*) Let (X, **T**) be a topological space, then prove the following :

(*i*) Subspace of first countable space is first countable space.

(*ii*) Subspace of second countable space is second countable space.

6. Answer the following : 20

(*a*) If (X, **T**) be a topological space, $A \subseteq X$, then prove that $\overline{A} = A \cup A'$.

(*b*) Show that product of Lindelof space need not be Lindelof space.

This question paper contains **3** printed pages]

# NEPSST—398—2025

## FACULTY OF ARTS/SCIENCE

## M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020 Pattern)

MATHEMATICS

Paper–SMATE-451 (A)

(Partial Differential Equations)

**(Friday, 25-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

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*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If $\bar{X}\, curl\, \bar{X} = 0$ where $X = P\bar{i} + Q\bar{j} + R\bar{k}$ and $\mu$ is an arbitrary differentiable function of $x$, $y$ and $z$, then prove that $u\bar{X}.curl\left(\mu\bar{X}\right) = 0$.

(*b*) Explain analytic expression for the Monge cone at $(x_0, y_0, z_0)$.

P.T.O.

(*c*) Explain classification of second order pde.

(*d*) Show that the solution of the Dirichlet problem if it exists is uniquae.

2. Answer the following : 20

(*a*) Let $u(x, y)$ and $v(x, y)$ be two functions of $x$ and $y$ such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u, v)}{\partial(x, y)} = 0$, then prove that there exist a relation between $u$ and $v$ not involving $x$ and $y$ explicitly.

(*b*) Find general integeral of $y^2p - xyq = x(z - 2y)$.

3. Answer the following : 20

(*a*) If $h_1 = 0$ and $h_2 = 0$ are compatible with $f(x, y, z, u_x, u_y, u_z) = 0$, then prove that $\frac{\partial(f, h)}{\partial(x, u_x)} + \frac{\partial(f, h)}{\partial(y, u_y)} + \frac{\partial(f, h)}{\partial(z, u_z)} = 0$, where $h = h_i (i = 1, 2)$.

(*b*) Find complete integral of $xpq + q^2y - 1 = 0$ by Charpit's method.

4. Answer the following : 20

(*a*) Reduce the equation $(n - 1)^2 u_{xx} - y^{2n} u_{yy} = ny^{2n-1} uy$ where $n$ is is an integer, to a canonical form.

(*b*) Derive canonical form for hyperbolic and elliptic type of pde.

5. Answer the following : 20

(*a*) State and prove Harnack theorem.

(*b*) Show that the solution for the Dirichlet problem for a circle is given by the Poisson integral formula.

6. Answer the following : 20

(*a*) Find complete integral of $p^2x + q^2y = z$ using Jacobi's method.

(*b*) Find the solution of the problem :

$\nabla^2 u = 0, -\infty < x < \infty, y > 0$

$u(x, 0) = f(x), -\infty < x < \infty$

Such that $u$ is bounded as $y \to \infty, u$ and $u_x$ vanishes as $|x| \to \infty$.

This question paper contains **4** printed pages]

# NEPSST—399—2025

## FACULTY OF ARTS/SCIENCE

## M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020 Pattern)

MATHEMATICS

Paper–SMATE-452

(Combinatorics)

**(Friday, 25-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) How many ways are there to distribute seven different books among 15 children if no child gets more than one book ?

(*b*) How many ways are there to select 25 toys from seven types of toys with between two and six of each type ?

P.T.O.

(*c*) Find a recurrence relation for number of ways to arrange $n$ distinct objects on a row. Find the number of arrangements of eight objects.

(*d*) How many arangements of the digits 0, 1, 2, ....., 9 are there in which the first digit is greater than 1 and the last digit is less than 8 ?

2. Answer the following : 20

(*a*) If there are $n$ objects, with $r_1$ of type 1, $r_2$ type 2, ....., and $r_m$ of type $m$, where $r_1 + r_2 + ..... + r_m = n$ then prove that the number of arrangements of these $n$ objects, denoted P(n; $r_1$, $r_2$ ....., $r_m$) is given by :

$$P(n; r_1, r_2, ....., r_m) = \binom{n}{r_1}\binom{n-r_1}{r_2}\binom{n-r_1-r_2}{r_3}...\binom{n-r_1-r_2-....-r_{m-1}}{r_m}$$

$$= \frac{n!}{r_1!\, r_2!\, ....r_m!}$$

(*b*) How many ways are there to form a three-letter sequence using the letters $a$, $b$, $c$, $d$, $e$, $f$ :

(*i*) with repetition of letters allowed ?

(*ii*) without repetition of any letter ?

(*iii*) without repetition and containing the letter $e$ ?

(*iv*) with repetition and containing $e$ ?

3. Answer the following : 20

(*a*) Build a generating function for $a_r$, the number of selections from :

(*i*) Five red, four black, and four white balls.

(*ii*) Five jelly beans, four licorcie, eight lollipops with at least one of each type of candy.

(*b*) Find the coefficient of $x^r$ in $(x^5 + x^6 + x^7 + .....)^7$.

4. Answer the following : 20

(*a*) An elf has a staircase of $n$ stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for $a_n$, the number of different ways for the elf to ascend the $n$-stair staircase.

(*b*) Find a recurrence relation for the number of ways to distribute $n$ distinct objects onto four boxes. What is the initial condition ?

5. Answer the following : 20

(*a*) Let $A_1, A_2, ....., A_n$ be $n$ sets in the universal set U. Then prove that :

$$N(A_1 \cup A_2 \cup .... \cup A_n) = S_1 - S_2 + S_3 - .... + (-1)^{k-1} S_k + .... + (-1)^{n-1} S_n$$

(*b*) Find the number of 4-digit ternary sequence with exactly two 1s. Also, find the number with at least two Is.

P.T.O.

6. Answer the following : 20

(*a*) In a bridge deal, what is the probability that :

(*i*) West has five spades, two hearts, three diamonds, and three clubs ?

(*ii*) North and South have five spades, West has two spades, and East has one spade ?

(*iii*) One player has all the aces ?

(*b*) What is the probability that a 8-card hand has at least one 4 of a kind ?

This question paper contains **4** printed pages]

# NEPSST—400—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020)

MATHEMATICS

Paper–SMATE-453

(Dynamics and Continuum Mechanics–II)

**(Friday, 25-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Let T be a tensor that transforms the specific vectors $\bar{a}$ and $\bar{b}$ as follows :

$$\bar{T}\,\bar{a} = a + 2b$$

$$\bar{T}\,\bar{b} = a - b$$

Give a vector $c = 2a + b$, then find $\bar{T}\,\bar{c}$ ?

P.T.O.

(*b*) Discuss curl of vector field.

(*c*) Derive an equation of conservation of mass.

(*d*) For linear isotropic Elastic Solid, prove that :

$$\mathrm{T}ij = \lambda_e\, \delta ij + 2\mu \mathrm{E}ij$$

2. Answer the following : 20

(*a*) Verify the following :

$$\in_{ijm} \in_{klm} = \delta_{ik}\ \delta_{jl} - \delta_{jl}\, \delta_{ik}$$

(*b*) Given $[\mathrm{S}ij] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 3 & 0 & 3 \end{bmatrix}$ and $[ai] = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Evaluate :

(*i*) $\mathrm{S}_{ii}$

(*ii*) $\mathrm{S}_{ij}$

(*iii*) $\mathrm{S}_{mn}\ a_m\ a_n$

3. Answer the following : 20

(*a*) Show that :

(*i*) $\dfrac{d}{dt}(\mathrm{T} + \mathrm{S}) = \dfrac{d\mathrm{T}}{dt} + \dfrac{d\mathrm{S}}{dt}$

(*ii*) $\dfrac{d}{dt}(\alpha(t)\mathrm{T}) = \dfrac{d\alpha}{dt}\mathrm{T} + \alpha\dfrac{d\mathrm{T}}{dt}$

(*iii*) $\dfrac{d}{dt}(\mathrm{T}^{\mathrm{T}}) = \left(\dfrac{d\mathrm{T}}{dt}\right)^{\mathrm{T}}$.

(*b*) Write a short note on Laplacian of vectors field.

4. Answer the following : 20

(*a*) Derive an expression to find Scalar invariants of tensor.

(*b*) The position at time $t$ of a particle initially at $(X_1, X_2, X_3)$ is given by the equations :

$$x_1 = X_1 + K (X_1 + X_2)t$$

$$x_2 = X_2 + k (X_1 + X_2)t$$

(*i*) Find the velocity at $t = 2$ for the particle that was at (1, 1, 0) at the reference time.

(*ii*) Find the velocity at $t = 2$ for the particle that is at position (1, 1, 0) at $t = 2$.

5. Answer the following : 20

(*a*) Discuss the principal stress.

(*b*) Show that for an incompressible fluid :

$$\frac{\partial T_{ij}}{\partial X_j} = \frac{\partial \rho}{\partial X_i} + \mu \frac{\partial^2 v_i}{\partial X_j \partial X_j}$$

6. Answer the following : 20

(*a*) Define Tensor and find its component.

P.T.O.

(*b*) Let $\phi$ and $\varphi$, be scalar fields $v$ and $w$ be vectors fields then verify the following identities :

(*i*) $\nabla(\phi + w) = \nabla\phi + \nabla w$

(*ii*) $div(\phi v) = (\nabla\phi)v + \phi(div.v)$

(*iii*) $div(v + w) = div\, v + div\, w$

(*iv*) $div(curl\, v) = 0$.

This question paper contains **4** printed pages]

# NEPSST—401—2025

## FACULTY OF SCIENCE

### M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

### APRIL/MAY, 2025

MATHEMATICS

(Operation Research)

**(Friday, 25-4-2025)** **Time : 10.00 a.m. to 1.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Explain the linear programming can be used for optimization problem.

(*b*) Give assumptions in the Transportation model.

(*c*) Define the Assignment model and give a suitable example for it.

(*d*) Write a short note on characteristics of games.

P.T.O.

2. Answer the following : 20

(*a*) Write a flow chart for the Simplex algorithm.

(*b*) Find the graphical method of solution : Maximum value of :

$$Z = 2x_1 + 3x_1$$

Subject to $x_1 + x_2 \leq 30,\ x_2 \geq 3,\ x_2 \leq 12,$

$$x_1 - x_2 \geq 0,\ 0 \leq x_1 \leq 20$$

3. Answer the following : 20

(*a*) Explain the North-West corner method.

(*b*) A Military equipment is to be transported from three origins to four destinuations. The supply at the origins, the demand at the destinations and time of shipment is shown in the table below. The units to be shipped as obtained by North-West courner rule are given in parenthese. Work at a transportation plan so that the total time required for shipment is minimum :

Destination

| Origins | 1 | 2 | 3 | 4 | $a_i$ |
|---|---|---|---|---|---|
| 1 | 10 (12) | 0 (3) | 20 | 11 | 15 |
| 2 | 1 | 7 (5) | 9 (15) | 20 (5) | 25 |
| 3 | 12 | 14 | 16 | 18 (5) | 5 |
| $b_j$ | 12 | 8 | 15 | 10 | 45 (Total) |

4. Answer the following : 20

(*a*) Explain Hungarian Assignment problems.

(*b*) Solve the following assignment problem :

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| 1 | 11 | 17 | 8 | 16 | 20 |
| 2 | 9 | 7 | 12 | 6 | 15 |
| 3 | 13 | 16 | 15 | 12 | 16 |
| 4 | 21 | 24 | 17 | 28 | 26 |
| 5 | 14 | 10 | 12 | 11 | 13 |

5. Answer the following : 20

(*a*) Explain the Travelling Salesman proam and write mathanatical statement of the Travelling salesman problem.

(*b*) A company has a team of four salesmen and four districts where the company wants to start its business. After taping into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in ruppes for each salesman in each district as below :

P.T.O.

| Salesmen | District 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 16 | 10 | 14 | 11 |
| B | 14 | 11 | 15 | 15 |
| C | 15 | 15 | 13 | 12 |
| D | 13 | 12 | 14 | 15 |

Find the assignment of salesman to various districts which will yield maximum profit.

6. Answer the following : 20

(*a*) Write assignment model makes use of two theorem on Transportation model.

(*b*) Products 1, 2, 3, 4 and 5 are to be processed on a machine. The setup cost in rupees per change depend upon the product presenty on the machine and the setup to be made and are given by the following data :

$C_{12}$ = 16, $C_{13}$ = 4, $C_{14}$ = 12, $C_{23}$ = 6, $C_{34}$ = 5, $C_{25}$ = 8, $C_{35}$ = 6, $C_{45}$, 20; $C_{ij} = C_{ji}$, $C_{ij} = \infty$ and value of $i$ and $j$.

Find the optimum sequence of products in order to minimize the toatl setup. cost.

This question paper contains **2** printed pages]

# NEPSST—296—2025

## FACULTY OF SCIENCE/ARTS

## MA/M.Sc. (Second Year) (Third Semester) EXAMINATION

## APRIL/MAY, 2025

MATHEMATICS

Paper-I

(Fluid Mechanics)

**(Thursday, 24-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B. :—* (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Explain strength of vortex tube.

(*b*) Write laplace equation in spherical polar co-ordinate system.

(*c*) Define stagnation point, real fluid, steady flow.

(*d*) Write a note on line sink.

2. Answer the following : 20

(*a*) Derive an equation of continuity for steady incompressible fluid.

(*b*) Illustrate local and particle rate of change.

P.T.O.

3. Answer the following : 20

(*a*) Derive Euler equation of motion.

(*b*) Explain the mechanism of Venturi meter.

4. Answer the following : 20

(*a*) Explain the motion of accelerating sphere moving in fluid at rest at infinity.

(*b*) State and prove Kelvin's theorem.

5. Answer the following : 20

(*a*) Find the equation of stream line due to sources of strength $m$ at the point A $(-c, 0)$ and B $(c, 0)$.

(*b*) Discuss the flow due to uniform line doublet at origin of strength $u$. Its axis being along $x$-axis.

6. Answer the following : 20

(*a*) Prove that, in fluid region the pressure is same in all direction.

(*b*) Discuss the nature of flow for an incompressible fluid $q = [-wy, wx, 0]$, where $w$ is constant.

This question paper contains **4** printed pages]

# NEPSST—298—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP 2020)

MATHEMATICS

SMATES-501

(Fuzzy Sets and Their Applications–I)

**(Thursday, 24-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time—Three Hours* *Maximum Marks*—80

*N.B.* :— (*i*) Question No. **1** is compulsory.

(*ii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iii*) *All* questions carry equal marks.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that for crisp set A and B :

$$\overline{A \cap B} = \bar{A} \cup \bar{B} \text{ and}$$

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

P.T.O.

(*b*) Prove that :

$$\lim_{w\to\infty} i_w = \lim_{w\to\infty} (1 - \min[1, (1-a)^w + (1-b)^w)^{1/w}])$$

$$= \min(a,\ b).$$

(*c*) Prove that :

$$\lim_{\alpha\to 0} h\alpha = (a_1, a_2, \ldots\ldots, a_n)^{1/n}$$

where, $h_\alpha(a_1, a_2, \ldots\ldots, a_n) = \dfrac{(a_1^\alpha + a_2^\alpha + \ldots\ldots + a_n^\alpha)^{1/\alpha}}{n}$.

(*d*) Given

$$\mathrm{Q} = \begin{bmatrix} .1 & .1 & .5 & 1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix} \quad \text{and}$$

$r$ = [.8, .7, .5, 0] find all solutions of PoQ = $r$ where

$$\mathrm{P} = [p_j \mid j \in \mathrm{J}],\ \mathrm{Q} = [q_{j,k} \mid j \in \mathrm{J}, k \in \mathrm{K}]$$

$$r = [r_k \mid k \in \mathrm{K}].$$

2. Answer the following : 20

(*a*) Justify your answer, whether the law of contradiction and the law of exclusive middle are valid in fuzzy set theory.

(*b*) Prove that, if a complement $c$ has an equilibrium $e_c$, then prove that $d_{e_c} = e_c$.

3. Answer the following : 20

(*a*) Prove that, $i(a, b) \leq \min(a, b)$ for all $a, b \in [0, 1]$.

(*b*) Show that the Sugeno complements are involved for all $X \in (-1, \infty)$.

4. Answer the following : 20

(*a*) Find the max-min composition and max-product composition for :

$$A = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix}.$$

(*b*) The fuzzy binary relation R is defined on sets X = {1, 2, 3, ....., 100} and Y = {50, 51, 52, ....., 100} and represents the relation "*x* is much smaller than *y*", defined by membership function :

$$\mu_R(x, y) = \begin{cases} 1 - \dfrac{x}{y}, & x \leq y \\ 0, & \text{otherwise} \end{cases}$$

where $x \in X$, $y \in Y$. Then find Range, domain and height of R.

5. Answer the following : 20

(*a*) Find the solution of matrix equation :

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix} \circ \begin{bmatrix} .9 & .5 \\ .7 & .8 \\ 1 & .4 \end{bmatrix} = \begin{bmatrix} .6 & .3 \\ .2 & 1 \end{bmatrix}$$

whose general form is :

$$\left[p_{ij}\right] \circ \left[q_{j,k}\right] = \left[r_{i,k}\right]$$

where $i \in \mathbf{N}_2$, $j \in \mathbf{N}_3$ and $k \in \mathbf{N}_2$.

P.T.O.

(*b*) Show that the equilibria $e_{c_w}$ for the Yager fuzzy complements are given by the formula :

$$e_{c_w} = \left(\frac{1}{2}\right)^{1/w}$$

Plot this function for $w \in (0, 10]$.

6. Answer the following : 20

(*a*) Define fuzzy partial ordering. Give examples of partial ordering in the form of Hasse diagrams.

(*b*) Does the function $c(a) = (1-a)^w$ qualify for each $w > 0$ as a fuzzy complement ? Plot the function for some values $w > 1$ and some values $w < 1$.

This question paper contains **4** printed pages]

# NEPSST—295—2025

## FACULTY OF ARTS/SCIENCE

## M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP-2020 Pattern)

MATHEMATICS

Paper SMATE-501(A)

(Integral Transforms)

**(Thursday, 24-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (1) *All* questions carry equal marks.

(2) Q. No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) State and prove the final value theorem for Laplace transform.

(*b*) Find the Mellin transform of :

$$f(x) = e^{-x}.$$

P.T.O.

(*c*) Find a Fourier integral representation of the function :

$$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(*d*) Find the Hankel transform of order $v$ of a function $f(r) = r^v h(a - r)$, $a > 0$, where $h$ is the Heaviside's unit function.

2. Answer the following : 20

(*a*) If $f$, $f'$, $f''$, ....., $f^{(n-1)}$ are all continuous functions on $t \geq 0$, $f^{(n)}$ is piecewise continuous on $t \geq 0$, and if all are of exponential order $c_0$, then for $n = 1, 2, 3, .....$, prove that :

$$\mathbf{L}[f^{(n)}(t)] = p^n \, \mathrm{F}(p) - p^{n-1} f(0) - p^{n-2} f'(0) - ..... - f^{(n-1)}(0).$$

where $\mathrm{F}(p)$ is the Laplace transform of $f(t)$.

(*b*) Find the inverse Laplace transform of the function :

$$\mathrm{F}(p) = \frac{2}{(p+1)(p^2+1)}.$$

3. Answer the following : 20

(*a*) Using Laplace transform, solve the initial value problem :

$$y'' - 6y' + 9y = t^2 e^{3t}, \quad y(0) = 2, \quad y'(0) = 6.$$

(*b*) Using Laplace transform, solve the following partial differential equation under the given boundary conditions (B.C.) and initial conditions (I.C.) :

$$u_{xx} = a^{-2}u_t, \qquad 0 < x < \infty,\ t > 0$$

B.C. : $u(0, t) = f(t), \qquad u(x, t) \to 0$ as $x \to \infty$

I.C. : $u(x, 0) = 0, \qquad 0 < x < \infty.$

4. Answer the following : 20

(*a*) If $f$ is piecewise continuous and absolutely integrable on the entire real axis, and if $x$ is a point of continuity of $f$, then prove that :

$$\lim_{\lambda \to \infty} \frac{1}{\pi} \int_{-\infty}^{\infty} f(x + t) \frac{\sin \lambda t}{t}\, dt = f(x).$$

(*b*) Using Fourier integral representations, show that :

(*i*) $$\int_0^{\infty} \frac{\sin s \cos s}{s}\, ds = \frac{\pi}{4}$$

(*ii*) $$\int_0^{\infty} \frac{\sin ax}{x}\, ds = \frac{\pi}{2}, \quad a > 0.$$

5. Answer the following : 20

(*a*) Using Fourier transform, solve the boundary value problem :

$$y'' + ay' - by = f(x), \quad -\infty < x < \infty$$

$$y(x) \to 0,\ y'(x) \to 0 \text{ as } |x| \to \infty,$$

where $a$ and $b$ ($b > 0$) are constants.

P.T.O.

(*b*) Using Fourier transform, solve the following partial differential equation under the given conditions :

$$u_{xx} = a^{-2}u_t, \qquad -\infty < x < \infty, \qquad t > 0$$

B.C. : $u(x, t) \to 0, \qquad u_x(x, t) \to 0$ as $|x| \to \infty$

I.C. : $u(x, 0) = f(x), \qquad -\infty < x < \infty.$

6. Answer the following : 20

(*a*) Using Laplace transform, solve the initial value problem :

$$y'' + 2y' + 5y = e^{-t} \sin t, \quad y(0) = 0, \quad y'(0) = 1.$$

(*b*) Derive the transform relations :

(*i*) $\mathbf{F}_c\{f''(t)\} = -s^2 \, \mathrm{F}_c(s) - \sqrt{\frac{2}{\pi}} \, f'(0)$

(*ii*) $\mathbf{F}_s\{f''(t)\} = -s^2 \, \mathrm{F}_s(s) - \sqrt{\frac{2}{\pi}} \, s \, f(0),$

where $\mathbf{F}_c\{f(t)\} = \mathrm{F}_c(s)$ is the Fourier cosine transform of $f(t)$ and $\mathbf{F}_s\{f(t)\} = \mathrm{F}_s(s)$ is the Fourier sine transform of $f(t)$.

This question paper contains **3** printed pages]

# NEPSST—15—2025

## FACULTY OF SCIENCE

### M.Sc. (Second Year) (Third Semester) EXAMINATION

### APRIL/MAY, 2025

MATHEMATICS

Paper SMATC 501

(Field Theory)

**(Wednesday, 16-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Show that the polynomial :

$f(x) = 1 + x + \ldots\ldots + x^{p-1}$ is irreducible over Q, where P is prime no.

(*b*) Construct the splitting field of the polynomial $x^3 - 1$ over Q.

(*c*) Prove that the group G (Q ($\alpha$)/Q), where $\alpha^5 = 1$ and $\alpha \neq 1$, is isomorphic to the cyclic group of order 4.

(*d*) Express the following symmetric polynomials as rational functions of the elementary symmetric functions :

$x_1^2 + x_2^2 + x_3^2$.

2. Answer the following : 20

(*a*) State and prove Gauss Lemma.

(*b*) Let $f \leq E \leq k$ be fields. If [K : E] < $\infty$

and [E : F] < $\infty$, then prove that :

(*i*) [K : F] < $\infty$

(*ii*) [K : F] = [K : E] [E : F]

3. Answer the following : 20

(*a*) Prove that any finite field F with $P^n$ elements is the splitting field of $x^{p^n} - x \in F_p [x]$ consequently any *two* finite field with $P^n$ elements are isomorphic.

(*b*) If $f(x) \in F[x]$ be a polynomial of degree $\geq 1$ with $\propto$ as a root, then prove that $\alpha$ is a multiple root if and only $f'(\alpha) = 0$.

4. Answer the following : 20

(*a*) If F and E are field, then prove that distinct embeddings of F into E are linearly independant over E.

(*b*) If E be a finite separable extension of a field F. Then prove that following are equivalents :

(*i*) E is a normal extension of F

(*ii*) F is fixed field of G (E/F).

5. Answer the following : 20

(*a*) If $a$ and $b$ are constructible numbers, then $a \pm b$ are also constructible.

(*b*) Prove that the following are equivalent statements :

(*i*) $a \in \mathrm{R}$ is constructible from Q

(*ii*) $(a, 0)$ is a constructible point from $\mathrm{Q} \times a$

(*iii*) $(a, a)$ is a constructible point from $\mathrm{Q} \times \mathrm{Q}$.

6. Answer the following : 20

(*a*) If F be a field and $n$ be a positive integer, then prove that these exists a primitive $n$th root of unity in some extension E of F if and only if either char F = 0 or char. F $\times$ $n$.

(*b*) If E be an extension field of F, and $u \in \mathrm{E}$ be algebraic over F and $\mathrm{P}(x) \in \mathrm{F}[x]$ be a polynomial of the least degree such that $\mathrm{P}(u) = 0$, then prove that :

(*i*) $\mathrm{P}(x)$ is irreducible over F

(*ii*) If $g(x) \in f(x)$ is such that $g(u) = 0$, then $p(x)/g(x)$.

(*iii*) These is exactly one monic polynomial $\mathrm{P}(x) \in \mathrm{F}(x)$ of least degree such that $\mathrm{P}(u) = 0$.

This question paper contains **3** printed pages]

# NEPSST—81—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

**M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP-2020)**

MATHEMATICS

Paper–SMATC-502

(Functional Analysis)

**(Saturday, 19-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that, vector addition and scalar multiplication are jointly continuous.

P.T.O.

(*b*) State and prove Pythagoras Theorem.

(*c*) If $T_1$ and $T_2$ are self-adjoint operators on Hilbert space H, then prove that $T_1.T_2$ is the self-adjoint operator on H if and only if $T_1.T_2 = T_2.T_1$.

(*d*) Define :

(*i*) Eigen Value

(*ii*) Eigen Vector

(*iii*) Eigen Space.

2. Answer the following : 20

(*a*) State and prove Hahn-Banach Theorem.

(*b*) State and prove the Open Mapping Theorem.

3. Answer the following : 20

(*a*) State and prove Schwartz's inequality.

(*b*) Prove that the closed convex subset C of a Hilbert space H contains a unique vector of the smallest norm.

4. Answer the following : 20

(*a*) If T* denotes the adjoint of an operator T, then prove the following :

(*i*) $(T_1 + T_2)^* = T_1^* + T_2^*$

(*ii*) $(T_1.T_2)^* = T_2^*.T_1^*$

(*iii*) $(\alpha \cdot T)^* = \overline{\alpha} \cdot T^*$, $\alpha$ – scalar

(*iv*) $\|T^*\| = \|T\|$.

(*b*) Define Unitary Operator. Prove that an operator T on Hilbert space H is unitary if and only if T is an isometric isomorphism of H onto itself.

5. Answer the following : 20

(*a*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, \ldots\ldots \lambda_m$ be the distinct eigen values of T corresponding to eigen vectors $x_1, x_2, \ldots\ldots x_m$. Suppose $M_1, M_2, \ldots\ldots M_m$ are corresponding eigen spaces and $P_1, P_2, \ldots. P_m$ are the projections on these eigen spaces. If $M_i$'s are pairwise orthogonal and spans H, then prove that, $P_i$'s are pairwise orthogonal, $I = \sum_{i=1}^{m} P_i$ and $T = \sum_{i=1}^{m} \lambda_i P_i$.

(*b*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, \ldots\ldots \lambda_m$ be the distinct eigen values of T corresponding to eigen vectors $x_1, x_2, \ldots\ldots x_m$. Suppose $M_1, M_2, \ldots\ldots M_m$ are corresponding eigen spaces and $P_1, P_2, \ldots. P_m$ are the projections on these eigen spaces.

If T is normal operator on H, then prove that, $M_i$'s are pairwise orthogonal.

6. Answer the following : 20

(*a*) State and prove Reisz Representation Theorem.

(*b*) Define normal operator. If N is normal operator on Hilbert space H, then prove that $\|N^2\| = \|N\|^2$.

This question paper contains **3** printed pages]

# NEPSST—177—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP 2020)

MATHEMATICS

SMATC-503

(Analytical Number Thoery)

**(Tuesday, 22-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

***N.B.*** *:—* (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt the following : 20

(*a*) Show that 41 divides $2^{20} - 1$.

(*b*) Verify that 3 is a primitive root of 7.

(*c*) Find the values of the following Legendre symbols :

(19/23) and (–23/59).

(*d*) Find all integer $n$ such that $\phi(n) = n/2$.

P.T.O.

2. Answer the following : 20

(*a*) State and prove Fermat's theorem.

(*b*) Solve $18x \equiv 30 \pmod{42}$.

3. Answer the following : 20

(*a*) If $p$ is a prime number and $d \mid p-1$, then prove that the congruence $x^d - 1 \equiv 0 \ (mod\ p)$ has exactly $d$ solutions.

(*b*) Find the order of integers 2, 3, 5.

(I) Modulo 17.

(II) Modulo 19.

4. Answer the following : 20

(*a*) State and prove quadratic reciprocity law.

(*b*) Solve the quadratic congruence : $5x^2 + 6x + 2 \equiv 0 \ (mod\ 13)$

5. Answer the following : 20

(*a*) Let $f$ be multiplicative, then prove that, if $f$ is completely multiplicative if and only if :

$$f^{-1}(n) = \mu(n) \cdot f(n), \text{ for all } n \geq 1.$$

(*b*) Define Mangoldt function $\Lambda(n)$. Find the values of Mangoldt function for $n = 1$ to 10.

6. Answer the following : 20

(*a*) Give an example to show that $a^2 \equiv b^2 \pmod{n}$ does not implies $a \equiv b \pmod{n}$. Also, find the four primitive root of 26.

(*b*) Determine whether the quadratic congruence is solvable or not $x^2 \equiv -46 \pmod{17}$ and also if $n \geq 1$, then prove that : $\log n = \sum_{d/n} \wedge(d)$.

This question paper contains **2** printed pages]

# NEPSST—43—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.Sc. (Second Year) (Fourth Semester) EXAMINATION

## APRIL/MAY, 2025

## (NEP 2020)

RESEARCH AND PUBLICATION ETHICS

NEPPE-1002

**(Thursday, 17-4-2025)** **Time : 2.00 p.m. to 4.00 p.m.**

*Time*—2 *Hours* *Maximum Marks*—40

***N.B.*** *:—* (*i*) Question Number **1** is compulsory.

(*ii*) Solve any *three* questions of the remaining.

1. Write notes on : 5×2=10

(*a*) Nature of philosophy

(*b*) Research integrity

(*c*) Importance of publication ethics

(*d*) Characteristics to call a journal open

(*e*) *h*-index.

P.T.O.

2. (*a*) Define philosophy. Explain its branches. 5×2=10

(*b*) What do you mean by fabrication, falsification and plagiarism (FFP).

3. (*a*) Define publication ethics. Write the importance of publication ethics. 5×2=10

(*b*) Describe SHERPA/ROMEO online resource and list three variant of text.

4. (*a*) What is predatory journal ? List the common characteristics of it. 5×2=10

(*b*) What is impact factor ? How does it calculate ? Explain it with a suitable example.

5. (*a*) What is plagiarism ? Give their types. 2×5=10

(*b*) Describe in detail SNIP and SJR.

6. Write short notes on : 4×2.5=10

(*a*) Moral philosophy

(*b*) Duplicate publication

(*c*) Principle of transparency

(*d*) Turnitin.

This question paper contains **4** printed pages]

# NEPSST—129—2025

## FACULTY OF SCIENCE AND TECHNOLOGY

## M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

## APRIL/MAY, 2025

MATHEMATICS

Paper SMATC-551

(Numerical Analysis)

**(Monday, 21-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (1) *All* questions carry equal marks.

(2) Q. No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

(5) Scientific calculator is allowed.

1. Answer the following : 20

(*a*) Perform two iterations of the Chebyshev method to obtain the approximate value of $\frac{1}{7}$. Take the initial approximation as $x_0 = 0.1$.

P.T.O.

(*b*) Show that the decomposition method fails to solve the system of equations :

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}.$$

(*c*) Determine the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

by using an iterative method. Given approximate inverse is :

$$B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}.$$

(*d*) Obtain the approximate value of sin(0.15) by using Lagrange interpolation. Use the data, sin(0.1) = 0.09983 and sin(0.2) = 0.19867.

2. Answer the following : 20

(*a*) Show that the Rate of Convergence of the Secant method is 1.618.

(*b*) Obtain the smallest positive root of the equation

$$f(x) = x^3 - 5x + 1 = 0$$

by Newton-Raphson method.

3. Answer the following : 20

(*a*) Explain in detail Gauss elimination method of solving the system of equations $Ax = b$.

(*b*) Solve the system of equations :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

by using the Cholesky method.

4. Answer the following : 20

(*a*) Discuss in detail successive over relaxation method of solving the system of equations $Ax = b$. Also obtain its error format.

(*b*) Solve the system of equations :

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

by using Jacobi iteration method. Take initial approximation as

$$x^{(0)} = [0.5 \quad -0.5 \quad -0.5]^T.$$

Perform three iterations.

5. Answer the following : 20

(*a*) Discuss in detail Lagrange interpolation, iterated interpolation and Newton's divided difference interpolation.

P.T.O.

(*b*) Construct the divided difference table for the data :

| $x$ | $f(x)$ |
|---|---|
| 0.5 | 1.625 |
| 1.5 | 5.875 |
| 3.0 | 31.0 |
| 5.0 | 131.0 |
| 6.5 | 282.125 |
| 8.0 | 531.0 |

Hence find the interpolating polynomial and also find the approximate value of $f(7)$.

6. Answer the following : 20

(*a*) Discuss in detail the Partition method to obtain the inverse of the matrix A.

(*b*) The function $f(x) = \sin x$ is defined on the interval [1, 3] :

(*i*) Obtain the Lagrange linear interpolating polynomial in the interval [1, 3] and find the bound on the truncation error. Obtain the approximate value of $f(1.5)$ and $f(2.5)$.

(*ii*) Divide the interval [1, 3] into two subintervals [1, 2] and [2, 3]. Obtain the Lagrange linear interpolating polynomial in each subinterval and find the bound on the truncation error. Find the approximate value of $f(1.5)$ and $f(2.5)$.

This question paper contains **3** printed pages]

# NEPSST—226—2025

**FACULTY OF SCIENCE AND TECHNOLOGY**

**M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP-2020 Pattern)**

MATHEMATICS

Paper–SMATC-522

(Classical Mechanics)

**(Wednesday, 23-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Write a short note on constraint with examples.

(*b*) If Lagrangian L is scleronomic, then prove that Hamiltonian H represents the total energy.

(*c*) Find the extremal of the functional :

$$J[y(x)] = \int_0^1 (y'^2 + 12xy)\,dx, \text{ with } y(0) = 0,\ y(\pi/2) = 1.$$

(*d*) Discuss the isoperimetric problems.

P.T.O.

2. Answer the following : 20

(*a*) Derive the Lagrange's equation of motion for conservative system.

(*b*) Find the Lagrangian L and equation of motion for simple pendulum. Find the time period T for simple pendulum.

3. Answer the following : 20

(*a*) Derive Hamilton's canonical equation of motion.

(*b*) Define Routh's function and obtain the Routh's equation of motion.

4. Answer the following : 20

(*a*) Prove that the necessary condition for $\int_{x_1}^{x_2} F(x,\ y,\ y')dx$ to be extremum is :

$$F_y - \frac{d}{dx} F_{y'} = 0$$

(*b*) Among all the curves joining two points $(x_0, y_0)$ and $(x_1, y_1)$. Find the one which generate the surface of minimum area when rotated about $x$-axis.

5. Answer the following : 20

(*a*) Find the plane curve of fixed perimeter and maximum area.

(*b*) Explain Brachstocrone problem and find its extremal.

6. Answer the following : 20

(*a*) Obtain an equation of motion in Linear Harmonic Oscillator. Also, prove that the generalised momentum conjugate to a cyclic co-ordinate is conserved.

(*b*) Discuss the invariance of Euler's equation. Also, find the extremal of the functional :

$$J\left[y(x)\right] = \int_0^{\pi/2} \left(y'^2 - y^2\right) dx \text{, with } y(0) = 0,\ y(\pi/2) = 1.$$

This question paper contains **4** printed pages]

# NEPSST—402—2025

**FACULTY OF ARTS/SCIENCE**

**M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION**

**APRIL/MAY, 2025**

**(NEP-2020 Pattern)**

MATHEMATICS

Paper SMATE 551(A)

(Integral Equations)

**(Friday, 25-4-2025)** **Time : 2.00 p.m. to 5.00 p.m.**

*Time*—3 *Hours* *Maximum Marks*—80

*N.B.* :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Convert the initial value problem into an integral equation :

$$y'' + xy = 1, \quad y(0) = 0, \quad y'(0) = 0$$

(*b*) Solve the integral equation :

$$y(x) = f(x) + \lambda \int_0^1 xt\ y(t)\ dt$$

P.T.O.

(*c*) Let the sequence $\{\phi_k(x)\}$ be all the eigen functions of a symmetric $\mathbf{L}_2$-kernel with $\{\lambda_k\}$ as the sequence of the corresponding eigen values. Then prove that the series $\sum_{n-1}^{\infty} \frac{|\phi_n(x)|^2}{\lambda_n^2}$ converges and its sum is bounded by $C_1^2$, which is an upper bound of the integral $\int_a^b |k^2(x, t)| dt$, where $k$ is the kernel of the Fredholm integral equation :

$$y(x) = f(x) + \lambda \int_a^b k(x, t)\, y(t)\, dt$$

(*d*) Using Laplace transform, find the resolvent kernel of the integral equation :

$$Y(t) = F(t) + \int_0^t (t - x)\, Y(x)\, dx$$

2. Answer the following : 20

(*a*) Reduce the following integral equation to a Fredholm integral equation. Conversely, recover the original differential equation with the initial conditions from the integral equation obtained.

$$y''(x) - \lambda y(x) = 0, \quad y(0) = 0, \quad y(l) = 0$$

(*b*) Convert the following integral equation to a Volterra integral equation. Conversely, derive the original differential equation with the initial conditions from the integral equation obtained :

$$y''(x) - 3y'(x) + 2y(x) = 4 \sin x, \quad y(0) = 1, \quad y'(0) = 2$$

3. Answer the following : 20

(*a*) Let $R(x, t; \lambda)$ be the resolvent kernel of the Fredholm integral equation :

$$y(x) = f(x) + \lambda \int_a^b k(x, t)\, y(t)\, dt.$$

Then prove that the resolvent kernel satisfies the integral equation :

$$R(x, t; \lambda) = k(x, t) + \lambda \int_a^b k(x, z)\, R(z, t; \lambda)\, dt.$$

(*b*) Show that the integral equation :

$$y(x) = f(x) + \frac{1}{\pi} \int_0^{2\pi} \sin(x + t)\, y(t)\, dt$$

possesses no solution for $f(x) = x$, but that it has infinitely many solutions when $f(x) = 1$.

4. Answer the following : 20

(*a*) Prove that the eigen values of a symmetric kernel of a Fredholm integral equation of the first kind are real.

(*b*) Prove that the multiplicity of any non-zero eigen value is finite for every symmetric kernel for which :

$$\int_a^b \int_a^b |k(x, t)|^2\, dx\, dt$$

is finite.

P.T.O.

5. Answer the following : 20

(*a*) Solve the general Abel singular integral equation :

$$f(x) = \int_0^x \frac{y(t)}{[h(x) - h(t)]^{\alpha}}\, dt, \quad 0 < \alpha < 1$$

where $h(x)$ is a strictly monotonically increasing and differentiable function known function in $(a, b)$ and $h(t) \neq 0$.

(*b*) Solve the equation :

$$f(x) = \int_0^x \frac{y(t)}{(\cos x - \cos t)^{\frac{1}{2}}}\, dt, \quad 0 \leq a < x < b \leq \pi$$

6. Answer the following : 20

(*a*) With the aid of resolvent kernel, find the solution of the Volterra integral equation :

$$y(x) = e^x \sin x + \int_0^x \frac{2 + \cos x}{2 + \cos t}\, y(t)\, dt.$$

(*b*) Using Laplace transform, solve the integral equation :

$$\int_0^t \frac{Y(x)}{\sqrt{t - x}}\, dx = 1 + t + t^2$$

and verify your solution.