This question paper contains **2** printed pages]

NEPWT—1001—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP-2020)

RESEARCH METHODOLOGY

(Tuesday, 10-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—60

N.B. :— (*i*) Question No. **1** is compulsory.

(*ii*) Of the remaining solve any *three* questions.

(*iii*) Calculator and log table is allowed.

1. Attempt any *three* of the following : 15

(*a*) Motivation in research

(*b*) Need for research designing

(*c*) ANOCOVA

(*d*) Statistical measure in research.

2. (*a*) What do you mean by research ? Describe the different steps involved in a research process. 8

(*b*) Discuss the observation method as a technique of data collection. 7

P.T.O.

3. (*a*) Calculate the mean, median and mode of the following data : 8

3, 6, 3, 7, 4, 3, 9

(*b*) Draw the flow diagram for hypothesis testing. 7

4. (*a*) What is Sampling ? Explain steps in sample design. 8

(*b*) Calculate the chi-square value of the following data : 7

Fully Agree	**Not Sure**	**Not Agree**	**Total**
102	108	75	285

5. (*a*) Define case study. Give their characteristics. 8

(*b*) Explain dependent and independent variables. 7

6. Write short notes on : 15

(*a*) Fundamental type of research

(*b*) Parametric test

(*c*) Secondary data sources.

This question paper contains **3** printed pages]

NEPWT—334—2024

FACULTY OF SCIENCE/ARTS

M.Sc./M.A. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(As per NEP 2020)

MATHEMATICS

Paper SMATE 401(A)

(Ordinary Differential Equations)

(Thursday, 19-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Solve $y''' - 3y' + 2y = 0$.

(*b*) Verify that $\phi_1(x) = x \ (x > 0)$ satisfies the equation $x^2 y'' - xy' + y = 0$ and find the second independent solution.

(*c*) Solve $x^2 y'' + 2xy' - 6y = 0$ for $x > 0$.

(*d*) Find all real valued solutions of $y' = \dfrac{e^{x-y}}{1 + e^x}$.

P.T.O.

2. Answer the following : 20

(*a*) Prove that the two solutions $\phi_1(x)$, $\phi_2(x)$ of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

(*b*) Solve $y'' + 4y' = \cos x$

3. Answer the following : 20

(*a*) $\phi_1, \phi_2 \ldots \phi_n$ be n solutions of $L(y) = 0$ on an interval I, and x_0 be any point in I, then prove that

$$W(\phi_1, \phi_2 \ldots\ldots \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)\, dt\right] W(\phi_1, \phi_2 \ldots \phi_n)(x_0)$$

(*b*) Derive two solutions of Legendre equation.

4. Answer the following : 20

(*a*) Prove that the two linearly independent solutions of $x^2 y'' + a(x)\, xy' + b(x)\, y = 0$ are $\phi_1(x) = |x|^{r_1} \sigma_1(x)$, $\phi_2(x) = |x|^{r_1 + 1} \sigma_2(x) + (\log |x|)\, \phi_1(x)$, where r_1, r_2 are the roots of indicial polynomial and a, b, σ_1, σ_2 have power series expansions which is convergent.

(*b*) Find two solutions of Euler's equation.

5. Answer the following : 20

(*a*) Prove that $M(x, y) + N(x, y)\,y' = 0$ is exact on some rectangle R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(*b*) A continuous real valued function on the rectangle R is defined by f and $|f(x, y)| \leq M$ for all (x, y) in R. Further f satisfies a Lipschitz condition with Lipschitz constant K in R. Prove that the successive approximations $\phi_0(x) = y_0$, $\phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))\,dt, (k = 0, 1, 2......)$ converge on the interval I : $|x - x_j| \leq \alpha = \min\{a, b/M\}$ to a solution ϕ of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on I.

6. Answer the following : 20

(*a*) Compute the Wronskian if characteristics polynomial of $L(y) = 0$ is $(r - r_1)^3$.

(*b*) Solve :

$$x^2 y'' + \frac{3}{2} xy' + xy = 0.$$

This question paper contains **3** printed pages]

NEPWT—335—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP 2020 Pattern)

MATHEMATICS

SMATE–401–(B)

(Discrete Mathematics)

(Thursday, 19-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

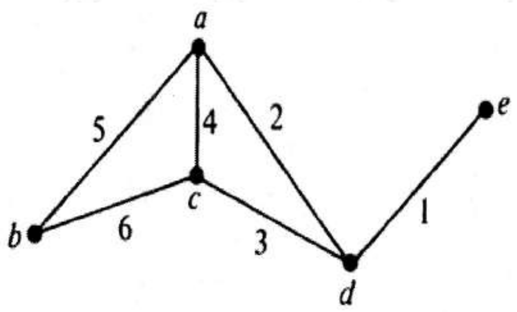
(*a*) Define a partially ordered set. Give an example of a partially ordered set which is not a lattice.

(*b*) Write a short note on Hamiltonian paths and circuits.

(*c*) Write a short note on centre of a tree with suitable examples.

P.T.O.

(*d*) Obtain an incidence matrix of the following graph.



2. Answer the following : 20

(*a*) Define the operations of join and meet in a lattice. Prove that the operations of join and meet in a lattice are commutative and associative.

(*b*) Define a complemented lattice with a suitable example. In a distributive lattice, if an element has a complement, then prove that this complement is unique.

Define a Boolean algebra with a suitable example. With usual notations, for any a and b in a Boolean algebra, prove that

$$\overline{a \vee b} = \bar{a} \wedge \bar{b} \text{ and } \overline{a \wedge b} = \bar{a} \wedge \bar{b}.$$

3. Answer the following : 20

(*a*) Define a path in graph with suitable example. Prove that if a graph (connected or disconnected) has exactly two vertices of odd degree, then there must be a path joining these two vertices.

(*b*) Write a short note on operations of graphs.

4. Answer the following : 20

(*a*) Define a tree with suitable examples. Prove that a tree with n vertices has $n - 1$ edges.

(*b*) Define a cut-set in a connected graph. Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.

5. Answer the following : 20

(*a*) Define incidence matrix of a graph. If A(G) is an incidence matrix of a connected graph G with n vertices, then prove that the rank of A(G) is $n - 1$.

(*b*) If B is a circuit matrix of a connected graph G with e edges and n vertices, then prove that rank of B = $e - n + 1$.

6. Answer the following : 20

(*a*) Define Hamiltonian circuits. Prove that in a complete graph with n vertices, there are $(n - 1)\backslash 2$ edge-disjoint Hamiltonian circuits if n is an odd number ≥ 3.

(*b*) Define a minimally connected graph. Prove that a graph is a tree if and only if it is minimally connected.

This question paper contains **3** printed pages]

NEPWT—336—2024

FACULTY OF SCIENCE

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper SMATE 401 C

(Dynamics and Continuum Mechanics-I)

(Thursday, 19-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt any *three* of the following (**5** marks each) : 20

(*a*) Write a short note on relative velocity and relative acceleration.

(*b*) Show that the total work done of a particle is the change in kinetic energy.

P.T.O.

(c) Show that the kinetic energy gained is $\frac{1}{2}\vec{I}\,(\vec{v}_1 + \vec{v}_2)$.

(*d*) A uniform rectangular lamina ABCD is such that AB = $2a$, BC = $2b$, then show that the direction of principal axes at A is $\tan 2\alpha = \frac{3ab}{2(a^2 - b^2)}$.

2. Attempt the following (**10** marks each) : 20

(*a*) Prove that the centroid of the system is unique.

(*b*) If $\vec{r} = [x, y, z]$ then show that :

(*i*) Div $\vec{r} = 3$

(*ii*) Curl $\vec{r} = 0$

(*iii*) Find the function $\phi\,(x, y, z)$ such that $\vec{r} = \text{grad}\,\phi$.

3. Attempt the following (**10** marks each) : 20

(*a*) Prove that the expression for vector angular velocity is $\vec{\omega} = \frac{1}{2}\,\text{curl}\,\vec{v}$.

(*b*) Prove that the kinetic energy of system of a particle moving generally in space is $\vec{T} = \frac{1}{2} M \dot{\vec{r}}^2 + \frac{1}{2}\sum m \dot{\vec{r}}^2$.

4. Attempt the following (**10** marks each) : 20

(*a*) With the usual notation prove that, the expression for the angular momentum of rigid body about a fixed point and fixed axis is $\vec{H} = I \times \vec{\omega}$.

(*b*) A uniform rigid rod AB moves so that A and B have velocities u_A, u_B at any instant. Show that the K.E. is then $T = \frac{1}{6} M\,(u_A^2 + u_A u_B + u_B^2)$.

5. Attempt the following (**10** marks each) : 20

(*a*) State and prove the principle of conservation of energy.

(*b*) Two particles of masses m_1 and m_2 at A and B connected by a rigid massless rod AB their velocities are $\vec{v}_1$ and $\vec{v}_2$ are suddenly changed by the application of externally impulses $\vec{J}_1$ and $\vec{J}_2$. Prove that the magnitude f of impulsive reaction of the rod on m_1 is

$$\frac{m_1.m_2}{m_1 + m_2} \hat{e} \left[\frac{\vec{J}_1}{m_1} - \frac{\vec{J}_2}{m_2} \right].$$

6. Attempt the following (**10** marks each) : 20

(*a*) Derive an expression for the velocity and acceleration for moving axis.

(*b*) Determine the moment of inertia of the distribution about the axis through O having direction cosines [λ, μ, ν] in terms of these direction cosines and A, B, C, D, E, F.

This question paper contains **5** printed pages]

NEPWT—337—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP-2020 Pattern)

MATHEMATICS

Paper SAMATE 401(D)

(Theory of Probability)

(Thursday, 19-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable calculator is allowed.

(*vi*) Use of table for area under standard normal curve is allowed.

1. Answer the following : 20

(*a*) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the bag at random. Find the probability that among the balls drawn, there is at least one ball of each colour.

P.T.O.

(*b*) If $p(x) = \begin{cases} \dfrac{x}{15}, & x = 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{cases}$

find :

(*i*) $P\ (x = 1 \text{ or } 2)$

(*ii*) $P\left(\dfrac{1}{2} < X < \dfrac{5}{2} \mid X > 1\right)$

(*c*) A and B play a game in which their chances of winning are in the ratio 3 : 2. Find A's chance of winning at least three games out of five games.

(*d*) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the arithmetic mean and the standard deviation of the distribution ?

2. Answer the following : 20

(*a*) State and prove the Boole's inequality for the probability of unions and intersections of n events A_1, A_2, ... A_n.

(*b*) (*i*) An integer is chosen at random from 1 to 200. What is the probability that the integer is divisible by 6 or 6 ?

(*ii*) Two dices are tossed. Find the probability of getting an even number on the first die or a total of 8.

3. Answer the following : 20

(*a*) Define the mathematical expectation of a discrete and continuous random variable. If X and Y are continuous random variables, then with usual notations, prove that :

$$E\ (X + Y) = E\ (X) + E\ (Y).$$

Hence or otherwise, prove that :

$$E\ (X_1 + X_2 + ... + X_n) = E\ (X_1) + E\ (X_2) + ... + E\ (X_n)$$

(*b*) (*i*) A random variable X is distributed at random between the values 0 and 1 so that its p.d.f. is $f\ (x) = kx^2\ (1 - x^3)$, where k is a constant. Find the value of k. Also, find its mean and variance.

(*ii*) A c.r.v. X has a p.d.f. $f\ (x) = kx^3\ (4 - x)^2$, $0 \leq x \leq 1$. Find k, the mean and the standard deviation of the distribution.

4. Answer the following : 20

(*a*) Define binomial distribution. Obtain first four moments about origin of the binomial distribution. Also obtain expressions for the first four central moments for binomial distribution.

P.T.O.

(*b*) A car hire firm has two cars, which it hires out day-by-day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which :

(*i*) neither car is used

(*ii*) some demand is refused.

(Take $e^{-1.5} = 0.2231$)

5. Answer the following : 20

(*a*) State the chief characteristics of the normal distribution and explain in detail the area property of the normal probability curve.

(*b*) The mean yield of one-acre plot is 662 kg with a standard deviation of 32 kg. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to have yield :

(*i*) over 700 kg.

(*ii*) below 650 kg.

What is the lowest yield of the best 100 plots ?

6. Answer the following : 20

(*a*) For n events $A_1, A_2, ..., A_n$, prove that :

(*i*) $P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1).\ P(A_2|A_1).\ P(A_3\ |A_1 \cap A_2)$ $...\ P(A_n\ |A_1 \cap A_2 \cap ... \cap A_{n-1})$

where $P(A_i\ |A_j \cap A_k \cap ... \cap A_l)$ represents the conditional probability of the event A_i given that the events $A_j, A_k, ..., A_l$ have already happened.

(*ii*) The events $A_1, A_2, ... A_n$ are mutually independent if and only if :

$$P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1).\ P(A_2).\ P(A_3)\ ...\ P(A_n).$$

(*b*) A sample of 100 items is taken at random from a batch known to contain 40% defectives. Assuming normal distribution, calculate the probability that the sample contains :

(*i*) at least 44 defectives

(*ii*) exactly 44 defectives.

This question paper contains **2** printed pages]

NEPWT—59—2024

FACULTY OF SCIENCE & TECHNOLOGY

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper SMATC-401

(Algebra)

(Thursday, 12-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that a group homomorphism $\phi : G \rightarrow H$ is injective if and only if $\ker(\phi) = \{e\}$.

(*b*) Prove that every finite group has a composition series.

(*c*) Prove that there is no simple group of order 81.

(*d*) Prove that the only homomorphism from ring of integers z to z are identity and zero mappings.

P.T.O.

2. Answer the following : 20

(*a*) Prove that every cyclic group of order n is isomorphic to $\mathbf{Z}_n$.

(*b*) State and prove 1st Isomorphism theorem.

3. Answer the following : 20

(*a*) If G be a nilpotent group, then every subgroup of G and every homomorphic image of G are nilpotent.

(*b*) Write down all the composition series for the $\mathbf{Z}_8$.

4. Answer the following : 20

(*a*) State and prove fundamental theorem of finitely generated abelian group.

(*b*) Find the non-isomorphic abelian group of order 16.

5. Answer the following : 20

(*a*) Prove that every euclidian domain is PID.

(*b*) Prove that an ideal M in the ring of integer Z is maximal ideal if and only if M = $\langle P \rangle$, P is prime number.

6. Answer the following : 20

(*a*) If H and K are normal subgroup of G and K $\subset$ H, then prove that $(G/K)/H/K \simeq G/H$.

(*b*) If R is a non-zero commutative ring with unity, then prove that an ideal M is a prime ideal if and only if R/M is an integral domain.

This question paper contains 4 printed pages]

NEPWT—125—2024

FACULTY OF SCIENCE & TECHNOLOGY

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP 2020)

MATHEMATICS

Paper SMATC-402

(Real Analysis)

(Saturday, 14-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If $f \in \mathbf{R}(\alpha)$ on $[a, b]$, then show that $f^2 \in \mathbf{R}(\alpha)$ on $[a, b]$.

(*b*) Let

$$f_n(x) = \frac{x^2}{\left(1 + x^2\right)^n}, \ (x \in \mathrm{R}, n = 0, 1, 2, ...).$$

Then prove that the series

$$\sum_{n=0}^{\infty} f_n(x)$$

of continuous functions converges to a discontinuous sum.

P.T.O.

(*c*) Does the every member of an equicontinuous family is uniformly continuous ? Justify your answer.

(*d*) Let $f : R^2 \to R^2$ be defined by the equation $f(x, y) = (e^x \cos y, e^x \sin y)$. Calculate D$f$ and $|Df|$.

2. Answer the following : 20

(*a*) Define rectifiable curve. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and

$$\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt.$$

(*b*) If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, show that $f \notin \mathbf{R}(\alpha)$ on $[a, b]$ for any $a < b$. Also, if $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then show that $fg \in \mathbf{R}(\alpha)$.

3. Answer the following : 20

(*a*) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that :

$$\lim_{t \to x} f_n(t) = A_n, \; n = 1, 2, 3, \ldots\ldots..$$

Then show that $\{A_n\}$ converges and :

$$\lim_{t \to x} f(t) = \lim_{t \to x} A_n.$$

(*b*) (*i*) For $m = 1, 2, 3, \ldots\ldots, n = 1, 2, \ldots\ldots$, let

$$S_{mn} = \frac{m}{m+n}.$$

Show that :

$$\lim_{n \to \infty} \lim_{m \to \infty} S_{mn} \neq \lim_{m \to \infty} \lim_{n \to \infty} S_{mn}.$$

(*ii*) Let

$$f_n(x) = \frac{\sin x}{\sqrt{n}}, \ (x \in \mathrm{R},\ n = 1, 2, \ldots\ldots),$$

then prove that

$$\lim_{n \to \infty} f_n'(x) \neq f'(x).$$

4. Answer the following : 20

(*a*) Suppose the series

$$\sum a_n x^n \text{ and } \sum b_n x^n$$

converge in the segment S = (–R, R). Let E be the set of all $x \in$ S at which :

$$\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n.$$

If E has a limit point in S, then prove that $a_n = b_n$ for $n = 0, 1, 2, \ldots\ldots\ldots$ hence holds for all $x \in$ S.

(*b*) If f is continuous on [0, 1] and if

$$\int_0^1 f(x) x^n dx = 0, n = 0, 1, 2, \ldots\ldots\ldots,$$

show that $f(x) = 0$ on [0, 1].

5. Answer the following : 20

(*a*) (*i*) Let A be open in R^n, let $f : \mathrm{A} \to \mathrm{R}^n$, let $f(\bar{b}) = \bar{a}$. Suppose that g maps a neighbourhood of $\bar{b}$ into R^n that $g(\bar{b}) = \bar{a}$ and $g(f(\bar{x})) = \bar{x}$ for all $\bar{x}$ in a neighbourhood of $\bar{a}$. If f is differentiable at $\bar{a}$ and if g is differentiable at $\bar{b}$, then prove that :

$$\mathrm{D}g(\bar{b}) = [\mathrm{D}f(a)]^{-1}.$$

P.T.O.

(*ii*) Let $A \subset R^m$, let $f : A \to R$. If f is differentiable at $\bar{a}$, then show that :

$$Df(\bar{a}) = [D_1 f(\bar{a}), D_2 f(\bar{a}), \ldots\ldots, D_m f(\bar{a})].$$

(*b*) Let $f : R^2 \to R^2$ be defined by the equation :

$$f(x, y) = (e^x \cos y, e^x \sin y)$$

(*i*) Show that f is one to one on the set A consisting of all (x, y) with $0 < y < 2\pi$.

(*ii*) What is the set $B = F(A)$?

If g is the inverse function, find $D_g(0, 1)$.

6. Answer the following : 20

(*a*) If P^* is a refinement of P, then prove that :

$$L(P, f, \alpha) \le L(P^*, f, \alpha)$$

$$\text{and } U(P^*, f, \alpha) \le U(P, f, \alpha).$$

Also, let $f_n(x) = n^2 x(1 - x^2)^n$, $0 \le x \le 1$ and $n = 1, 2, 3, \ldots\ldots$, show that :

$$\lim_{n \to \infty}\left[\int_0^1 f_n(x)\,dx\right] \ne \int_0^1 \left[\lim_{n \to \infty} f_n(x)\right] dx.$$

(*b*) State the Stone-Weierstrass theorem. Define $f : R^2 \to R$ by setting $f(\bar{0}) = \bar{0}$ and $f(x, y) = \dfrac{x^2}{x^4 + y^2}$, if $f(x, y) \ne \bar{0}$. Show that all directional derivative of f exist to $\bar{0}$ but that f is not differentiable at $\bar{0}$.

This question paper contains **3** printed pages]

NEPWT—192—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper–SMATC-403

(Complex Analysis)

(Tuesday, 17-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find all the values of z such that $e^{3z} = 1$.

(*b*) Show that the function $u(x, y) = ax + by$ is harmonic. Also determine its harmonic conjugate.

P.T.O.

(*c*) Evaluate $\int_C \bar{z}\,dz$ where $C : z(t) = e^{2it}\ (-\pi \le t \le \pi)$.

(*d*) Find the principal part of the Laurent's expansion for the function $f(z) = \dfrac{z}{z^2+4}$ valid in the neighborhood of $z = -2i$.

2. Answer the following : 20

(*a*) Define Cross Ratio. Prove that Cross ratio is invariant under the bilinear transformation.

(*b*) Find all the values of z such that $e^z = a + ib$.

Further, find all the values of z such that $e^z = 5 - 5i$.

3. Answer the following : 20

(*a*) Define Contour. State and prove Cauchy's main theorem.

(*b*) Find the values of a, b and c such that the following function, $f(z) = a(x^2 + y^2) + ibxy + c$ is an entire function.

4. Answer the following : 20

(*a*) State and prove Liouville's theorem.

(*b*) Evaluate :

(*i*) $\displaystyle\int_C \frac{3z^4 + 2z - 6}{(z-2)^3}\,dz$ where, $C : |z| = 3$

(*ii*) $\displaystyle\int_C \frac{e^z \sin z}{(z-2)^2}\,dz$ where, $C : |z| = 3$.

5. Answer the following : 20

(*a*) Define Argument Principle. State and prove Rouche's theorem.

(*b*) (*i*) Find [Res : $f(z)$; $z = 1$] for the function $f(z) = \dfrac{z^4 - z^3 + 17z + 12}{(z-1)^3}$.

Also evaluate $\int_C f(z)\, dz$.

(*ii*) Evaluate $\int_C \dfrac{1}{z(z-3)}\, dz$ along any simple closed contour C.

6. Answer the following : 20

(*a*) Define Harmonic function.

Show that the function $u(x, y) = e^{-x} \cos y + x$ is harmonic.

Also find the harmonic conjugate of $u(x, y)$ and all the analytic functions $f(z)$ such that $\mathrm{Re}(f(z)) = u(x, y)$.

(*b*) State and prove Casorati-Weierstrass' theorem.

This question paper contains **3** printed pages]

NEPWT—293—2024

FACULTY OF SCIENCE/ARTS

M.Sc./M.A. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(As per NEP 2020)

MATHEMATICS

Paper SMATE 451 (A)

(Partial Differential Equations)

(Wednesday, 18-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find Complete integral of $p + q - pq = 0$.

(*b*) Write a note on characteristic strip.

P.T.O.

(*c*) Derive canonical form for hyperbolic type of equations.

(*d*) Prove that the solution of Neumann problem is either unique or it differs from one another by a constant only.

2. Answer the following : 20

(a) Prove that the necessary and sufficient condition for the integrability of $dz = \phi(x, y, z)\, dx + \psi(x, y, z)\, dy$ is $[f, g] = 0$ where $f(x, y, z, p, q) = 0$, $g(x, y, z, p, q) = 0$.

(*b*) Show that the equations $xp - yq - x = 0, x^2 p + q - xz = 0$ are compatible and find a one parameter family of common solutions.

3. Answer the following : 20

(*a*) Solve $xu_x + yu_y = u_z^2$ by using Jacobi's method.

(*b*) Find the integral surface of the partial differential equation $(p^2 + q^2)x = pz$, passing through the curve C : $x_0 = 0$, $y_0 = s^2$, $z_0 = 2s$.

4. Answer the following : 20

(*a*) Find an expression of d'Alembert's solution which describes the vibrations of an infinite string.

(*b*) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to a canonical form.

5. Answer the following : 20

(*a*) Suppose that $u(x, y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$, then prove that u attains its maximum on the boundary B of D.

(*b*) Solve the equation $u_{xx} + u_{yy} = 0$, $0 \le x \le a$, $0 \le y \le b$, subject to the boundary conditions $u_x(0, y) = u_x(a, y) = 0$, $u_y(x, 0) = 0$, $u_y(x, b) = f(x)$.

6. Answer the following : 20

(*a*) Show that the necessary condition for the existence of the solution of the Neumann problem $\nabla^2 u = 0$ in D and $\frac{\partial u}{\partial n} = f(s)$ on B, is that integral of f over the boundary B should vanish. Here B is the boundary of the region D.

(*b*) Find complete integral of $p^2 + q^2 = x + y$.

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NEPWT—41—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper–SMATC-451

(Linear Algebra)

(Wednesday, 11-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : (5 marks each)

(*a*) Let V be a vector space over field F and $S_1 \leq S_2 \leq V$. If S_1 is linearly dependent, then prove that S_2 is also linearly dependent.

(*b*) If $T : \mathbf{R}^2 \to \mathbf{R}^3$, and $U : \mathbf{R}^2 \to \mathbf{R}^3$ are linearly transformation defined by $T(a_1, a_2) = (a_1 + 3a_2, 2a_1 - 4a_2)$

$U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$.

If β and ν are standard bases for $\mathbf{R}^2$ and $\mathbf{R}^3$ respectively, then find $[T]_\beta^\nu, [U]_\beta^\nu$ and $[T + U]_\beta^\nu$.

P.T.O.

(*c*) If $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$, then find all eigen values and corresponding eigen vectors of A.

(*d*) If $V = M_{n \times n}(F)$ and (,) : V. $xy \rightarrow F$ defined by $(A, B) = tr\ (B^*. A)$, then prove that the function (,) be an inner product on V.

2. Answer the following : (10 marks each)

(*a*) If W is a subspace of a finite dimensional vector space V, then prove that, basis for W can be extended to a basis for V.

(*b*) Prove that span of any subset S of a vector space V is a subspace of V. Moreover, any subspace of V that contains S must also contain span of S.

3. Answer the following : 10 marks each

(*a*) State and prove dimension theorem.

(*b*) Let V and W are finite dimensional vector space with ordered bases β and ν respectively. If T, U : $V \rightarrow W$ are linear transformation, then prove that :

(*i*) $[T + U]_\beta^\nu = [T]_\beta^\nu + [U]_\beta^\nu$

(*ii*) $[aT]_\beta^\nu = 9 \quad [T]_\beta^\nu$; $a \in F$.

4. Answer the following : 10 marks each

(*a*) If $Ax = b$ be the system of linear equation, then prove that system $A_x = b$ is consistent iff rank (A) = rank $(A|b)$.

(*b*) Prove that characteristic polynomial of any diagonalizable linear operator splits.

5. Attempt the following : 10 marks each

(*a*) Let V be an inner product space over F. If $x, y \in$ V and $c \in$ F, then prove that :

(*i*) $\|(x)\| = |c| \| x \|$

(*ii*) $\| x + y \| \leq \| x \| + \| y \|$.

(*b*) If T and U are linear operators on inner product space V, then prove that :

(*i*) $(T + U)^* = T^* + U^*$

(*ii*) $(TU)^* = U^* T^*$.

6. Answer the following : (10 makrs each)

(*a*) Let V and W are vector spaces of equal (finite) dimension. If T : V $\rightarrow$ W be a linear transformation, then prove that the following are equivalent :

(*i*) T is one-one

(*ii*) T is onto.

(*b*) Let T be a linear operator on a n-dimensional vector space V. If T has n-distinct eigen values, then prove that T is diagonalizable.

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NEPWT—294—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP 2020 Pattern)

MATHEMATICES

Paper SMATE-452

(Combinatorics)

(Wednesday, 18-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

Note :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) How many ways are there to rank n candidates for the job of chief wizard ? It the ranking is made at random (each ranking is equally likely), what is the probability that the fifth candidate Gandalf, is in second place ?

(*b*) Find a generating function for the number of ways to select r doughnuts from five chocolate, five strawberry, three lemon, and three cherry doughnuts. Repeat with the additional constraint that there must be at least one of each type.

P.T.O.

(*c*) Find a recurrence relation for number of ways to arrange n distinct objects on a row. Find the number of arrangements of eight objects.

(*d*) How many positive integers $\leq$ 70 are relatively prime to 70 ?

2. Answer the following : 20

(*a*) Prove that the number of ways to distribute r identical balls into n distinct boxes with at least one ball in each box is $c(r-1, n-1)$. With at least r_1 balls in the first box, at least r_2 balls in the second box ...., and at least r_n balls in the nth box, prove that the number is :

$$C(r - r_1 - r_2 - \dots - r_n + n - 1, n - 1).$$

(*b*) Nine students, three from Ms. A's class, three from Mr. B's class, and three from Ms. C's class have bought a block of nine seats for their school's homecoming game. If three seats are randomly selected for each class from the nine seats in a row, what is the probability that the three A students, three B students, and three C students will each get a block of three consecutive seats ?

3. Answer the following : 20

(*a*) Find a generating function for a_r, the number of ways to distribute r identical objects into five distinct boxes with an even number of objects not exceeding 10 in the first two boxes and between three and five in the other boxes.

(*b*) How many ways are there to distribute 25 identical balls into 7 distinct boxes if first box can have no more than 10 balls but any number can go into each of the other six boxes ?

4. Answer the following : 20

(*a*) Suppose we draw n straight lines on a piece of paper so that every pair of lines intersect (but no three lines intersect at a common point). Into how many regions do these n lines divide the plane.

(*b*) A bank pays 4 percent interest each year on money in savings accounts. Find recurrence relations for the amounts of money a gnome would have after n years if it follows the investment strategies of :

(*i*) Investing Rs. 1,000 and leaving it in the bank for n years.

(*ii*) Investing Rs. 100 at the end of each year.

5. Answer the following : 20

(*a*) Let A_1 A_2, ..., A_n be n sets in the universal set U. Then prove that :

$$N(A_1 \cup A_2 \cup \ldots \cup A_n) = S_1 - S_2 + S_3 - \ldots + (-1)^{k-1} S_k + \ldots + (-1)^{n-1} S_n$$

(*b*) How many m-digit decimal sequences (using digits 0, 1, 2, ..., 9) are there in which digits 1, 2, 3 all appear ?

P.T.O.

6. Answer the following : 20

(*a*) In a bridge deal, what is the probability that :

(*i*) West has five spades, two hearts, three diamonds, and three clubs.

(*ii*) North and South have five spades, West has two spades, and East has one spade.

(*iii*) One player has all the aces.

(*b*) How many ways are there to colour the four vertices in the graph shown in the following figure with n colours such that vertices with a common edge must be different colours.

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NEPWT—107—2024

FACULTY OF SCIENCE & TECHNOLOGY

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper SMATC-452

(Measure and Integration Theory)

(Friday, 13-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—**3** *Hours* *Maximum Marks*—**80**

N.B. :— (1) *All* **questions carry equal marks.**

(2) **Question No. 1 is compulsory.**

(3) **Answer any** *three* **questions from Q. No. 2 to Q. No. 6.**

(4) **Figures to the right indicate full marks.**

1. **Answer the following :** **20**

(*a*) **Show that if F $\in$ M and $m^*(F \Delta G) = 0$, then G is measurable.**

(*b*) **Show that if $f, g \in$ BV $[a, b]$, then $f \cdot g \in$ BV $[a, b]$.**

(*c*) **Write a short note on Hereditary class.**

(*d*) **Show that :**

$$\nu^+ = \frac{1}{2}(\nu + |\nu|), \; \nu^- = \frac{1}{2}(|\nu| - \nu),$$

provided ν is finite valued.

P.T.O.

2. Answer the following : 20

(*a*) Prove that, the class M is a σ-algebra.

(*b*) Show that every countable set has measure zero. Also show that if f and g are measurable, $|f| \leq |g|$ a.e., and g is integrable, then f is integrable.

3. Answer the following : 20

(*a*) Let $f \in BV\ [a, b]$, then prove that : $f(b) - f(a) = P - N$ and $T = P + N$, where all variations being in the finite interval $[a, b]$.

(*b*) Let f be defined by $f(x) = x \sin(1/x)$ for $x \neq 0$, $f(0) = 0$, find the four derivatives at $x = 0$.

4. Answer the following : 20

(*a*) Let μ^* be an outer measure on H(R) and let S^* denote the class of μ^* measurable sets. Then prove that S^* is a σ-ring and μ^* restricted to S^* is a complete measure.

(*b*) Show that :

$$H(R) = \left\{E!\ E \subseteq \bigcup_{n=1}^{\infty} E_n, E_n \in R\right\}.$$

5. Answer the following : 20

(*a*) State and prove Hahn decomposition theorem.

(*b*) Show that the following conditions on the measures μ and ν on $\lceil[X, S]\rceil$ are equivalent :

(*i*) $\nu << \mu$

(*ii*) $|\nu| << |\mu|$

(*iii*) $\nu^+ << \mu$ and $\nu^- << \mu$.

6. Answer the following : 20

(*a*) Show that, the constant functions are measurable. Also, if $f \in L(a, b)$, then prove that and $f \in$ BE $[a, b]$.

(*b*) (*i*) Let $f = g$ *a.e.* (μ), where μ is a complete measure. Show that if f is measurable, so is g.

(*ii*) Show that, a countable union of sets positive with respect to a signed measure ν is positive set.

This question paper contains 3 printed pages]

NEPWT—174—2024

FACULTY OF SCIENCE & TECHNOLOGY

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP 2020)

MATHEMATICS

Paper SMATC-453

(Topology)

(Monday, 16-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (1) *All* questions carry equal marks.

(2) Question No. **1** is compulsory.

(3) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(4) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If X is a non-empty set, then show that $T_f = \{A \subseteq X : A^c \text{ is finite or } X\}$ is a topology on X.

(*b*) If A is subspace of X, then show that the inclusion mapping $j : A \to X$ is continuous function.

(*c*) Show that the image of compact space under continuous map is compact.

(*d*) Show that the set of real numbers with usual topology is normal space.

P.T.O.

2. Answer the following : 20

(*a*) Define closed set. If (X, T) is a topological space, prove the following :

(*i*) ϕ, X are closed

(*ii*) Arbitrary intersection of closed sets is closed set.

(*iii*) Finite union of closed set is closed.

(*b*) Define Basis of the topology. If $\mathbf{B}'$ denotes the collection of all rectangular regions whose sides are parallel to the coordinate axes. $\mathbf{B}$ denotes the collection of all circular region, then show that topologies generated by $\mathbf{B}$ and $\mathbf{B}'$ are same.

3. Answer the following : 20

(*a*) If X and Y are topological spaces, then prove that the following conditions are equivalent :

(*i*) $f : X \to Y$ is continuous.

(*ii*) If V is closed in Y, then $f^{-1}(V)$ is closed in X.

(*b*) Prove that image of connected set under continuous function is connected set.

4. Answer the following : 20

(*a*) If X is compact and Y is Hausdorff space and $f : X \to Y$ is bijective continuous function, then prove that f is homeomorphism.

(*b*) Prove that every compact topological space (X, T) is limit point compact.

5. Answer the following : 20

(*a*) Prove that closed subspace of normal space is normal.

(*b*) Define first countable space. Prove that every second countable space is first countable space.

6. Answer the following : 20

(*a*) If X and Y are topological spaces, then prove that the following conditions are equivalent.

(*i*) $f : X \to Y$ is continuous.

(*ii*) If V is closed in Y, then $f^{-1}(V)$ is closed in X.

(*b*) Prove that subspace of regular space is regular space.

This question paper contains 4 printed pages]

NEPWT—296—2024

FACULTY OF SCIENCE/ARTS

MA/M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper-SMATE-454

(Operation Research)

(Wednesday, 18-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

Note :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Define feasible solution, optimal solution.

(*b*) Explain post optimality analysis in transportation.

(*c*) Write mathematical representation of assignment model.

(*d*) Define competitive game.

P.T.O.

2. Answer the following : 20

(*a*) Show by Simplex method, that the LPP has infinite number of non-basic feasible optimal solutions :

Maximize $Z = 4x_1 + 10\,x_2$

Subject to
$$2x_1 + x_2 \le 10$$
$$2x_1 + 5x_2 \le 20$$
$$2x_1 + 3x_2 \le 18$$
$$x_1, x_2 \ge 0$$

(*b*) Explain steps involved in the solution of two-phase method.

3. Answer the following : 20

(*a*) Explain formulation and solution of transportation models.

(*b*) Define transportation model and solve

2	3	11	7
1	0	6	1
5	8	15	9

by Vogel's approximation method.

4. Answer the following : 20

(*a*) A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1, 2, 3, 4, 5 and 6. The distance between the cities in kilometres is shown in the matrix below.

Find the assignment of trucks from cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum :

	1	2	3	4	5	6
A	12	10	15	22	17	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

(*b*) Solve the minimal assignment problem whose effectiveness matrix is :

2	3	4	5
4	5	6	7
7	8	9	8
3	5	8	4

5. Answer the following : 20

(*a*) For any 2 × 2 two person zero-sum game without any saddle point, having payoff matrix for player A as :

Player B

	B_1	B_2
A_1	a_{11}	a_{12}
A_2	a_{21}	$a22$

Find the optimal mixed strategies and value of the game using algebraic method.

P.T.O.

(*b*) Reduce the following game by dominance and find the game value :

Player B

	1	2	3	4	5
I	1	3	2	7	4
II	3	4	1	5	6
III	6	5	7	6	5
IV	2	0	6	3	1

6. Answer the following : 20

(*a*) Explain post optimality analysis in transportation problem.

(*b*) Explain the travelling salesman problem

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NEPWT—205—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP-2020)

MATHEMATICS

Paper–SMATE-501(A)

(Integral Transforms)

(Tuesday, 17-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) State and prove the initial value theorem for Laplace transform.

(*b*) Find the Mellin transform of $1/(1 + ax)^m$, $m > 0$.

(*c*) Find the Fourier transform of $f(t) = e^{-a|t|}$, $a > 0$.

(*d*) Find the Hankel transform of order v of a function $f(r) = r^{v-1}e^{-ar}, a > 0$.

P.T.O.

2. Answer the following : 20

(*a*) If f is piecewise continuous on $t \geq 0$ and is $0(e^{c_0 t})$, then prove that :

$$£\ [t^n f(t)] = (-1)^n\ F^{(n)}(p)$$

where F(p) is the Laplace transform of $f(t)$.

(*b*) Find the inverse Laplace transform of the function $F(p) = \frac{5p-2}{3p^2+4p+8}$.

3. Answer the following : 20

(*a*) Using Laplace transform, evaluate the integral :

$$\int_0^\infty \frac{\cos tx}{x^2+1}\, dx, \quad t > 0$$

(*b*) Using Laplace transform, solve the following partial differential equation under the given boundary conditions (B. C.) and initial conditions (I. C.) :

$$u_{xx} = c^{-2} u_{tt}, \qquad 0 < x < \infty, \qquad t > 0$$

$$\text{B.C. : } u(0, t) = f(t), \quad u(x, t) \to 0 \text{ as } x \to \infty$$

$$\text{I.C. : } u(x, 0) = 0, \quad u_t(x, 0) = 0, \quad 0 < x < \infty$$

4. Answer the following : 20

(*a*) If f is piecewise continuous and absolutely integrable on the entire real axis, then prove that :

$$\lim_{\lambda \to \infty} \int_{-\infty}^{\infty} f(t) \cos \lambda t\, dt = \lim_{\lambda \to \infty} \int_{-\infty}^{\infty} f(t) \sin \lambda t\, dt = 0.$$

or equivalently,

$$\lim_{\lambda\to\infty} \int_{-\infty}^{\infty} f(t)\, e^{i\lambda t}\, dt = 0.$$

(*b*) Find a Fourier cosine and Fourier sine integral representation of the function :

$$f(x) = \begin{cases} \cos x, & 0 < x < \dfrac{\pi}{2} \\ 0, & x > \dfrac{\pi}{2} \end{cases}.$$

5. Answer the following : 20

(*a*) Use Fourier transform to solve :

$$y'' - y = -h(1 - |x|), \quad -\infty < x < \infty$$

$$y(x) \to 0,\ y'(x) \to 0 \ \text{ as } \ |x| \to \infty,$$

where h is the Heaviside unit function.

(*b*) Using Fourier transform, solve the boundary value problem :

$$y'' - y = e^{-x}, \ 0 < x < \infty$$

$$y'(0) = 0,\ y(x) \to 0,\ y'(x) \to 0 \ \text{ as } \ x \to \infty$$

6. Answer the following : 20

(*a*) Find the inverse Laplace transform of the function $F(p) = \dfrac{(p+1)e^{-\pi p}}{p^2 + p + 1}$.

P.T.O.

(*b*) If $F\{f(t)\} = F(s)$ is the Fourier transform of $f(t)$, $FC\{f(t)\} = F_C(s)$ is the Fourier cosine transform of $f(t)$ and $F_s\{f(t)\} = F_s(s)$ is the Fourier sine transform of $f(t)$, then prove that :

(*i*) $F_C\{f(at)\} = \left(\frac{1}{a}\right) F_C\left(\frac{s}{a}\right),\ a > 0$

(*ii*) $F_s\{f(at)\} = \left(\frac{1}{a}\right) F_s\left(\frac{s}{a}\right),\ a > 0$.

This question paper contains **3** printed pages]

NEPWT—208—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (NEP) (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

SMATE-501

(Fuzzy Sets and their Applications-I)

(Tuesday, 17-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that for crisp set A and B $\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

(*b*) If C is a continuous fuzzy complement, then prove that c has a unique equilibrium.

(*c*) Prove that $\lim_{\alpha \to 0} h_\alpha = (a_1 . a_2 \dots a_n)^{1/n}$ where

$$h_\alpha(a_1, a_2, \dots, a_n) = \left(\frac{a_1^\alpha + a_2^\alpha + \dots + a_n^\alpha}{n} \right)^{1/\alpha}$$

(*d*) Define fuzzy relation equations with a suitable example.

P.T.O.

2. Answer the following : 20

(*a*) Define fuzzy sets with suitable examples why we need fuzzy set theory.

(*b*) Prove that, if a complement c has an equilibrium e_c, then prove that $d_{e_c} = e_c$.

3. Answer the following : 20

(*a*) Prove that :

$$\lim_{\omega \to \infty} \min [1, (a^{\omega} + b^{\omega})^{1/\omega}] = \max (a, b)$$

(*b*) For all $a, b \in [0, 1]$, $i\,(a, b) \leq \min\,(a, b)$.

4. Answer the following : 20

(*a*) Let R (X, Y) be a fuzzy relation on X = $\{x, y, z\}$ and Y = $\{a, b\}$ such that $M_R = \begin{array}{c} \\ x \\ y \\ z \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} .3 & .2 \\ 0 & 1 \\ .6 & .4 \end{bmatrix} \end{array}$, then find inverse of R (X, Y) and μ_0 Max-min composition product of them.

(*b*) The transitive max-min closure R_T (X, X) for a fuzzy relation R (X, X) defined by the membership matrix $M_R = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$ .Then find $M_{RU\ (ROR)}$.

5. Answer the following : 20

(*a*) Draw pictorial presentation of ordinary homomorphism and strong fuzzy homomorphism.

(*b*) Find the solution of matrix equation :

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix} \circ \begin{bmatrix} .9 & .5 \\ .7 & .8 \\ 1 & .4 \end{bmatrix} = \begin{bmatrix} .6 & .3 \\ .2 & 1 \end{bmatrix}$$

whose general form is :

$[p_{i,j}] \circ [q_{j,k}] = [r_{i,k}]$ where $i \in \mathbf{N}_2$, $j \in \mathbf{N}_3$ and $k \in \mathbf{N}_2$.

6. Answer the following : 20

(*a*) Given :

$$Q = \begin{bmatrix} .1 & .4 & .5 & 1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix} \text{ and}$$

$r = [.8, .7, .5, 0]$ determines all solutions for PoQ $= r$, where

$P = \{P_j | j \in J\}$, $Q = [q_j, k | j \in J, k \in k]$

$r = [rk \mid k \in k]$.

(*b*) Does the function $c(a) = (1 - a)^w$ qualify for each $w > 0$ as a fuzzy complement ? Plot the function for some values $w > 1$ and some values $w < 1$.

This question paper contains **2** printed pages]

NEPWT—230—2024

FACULTY OF SCIENCE

M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP-2020)

PHYSICS

Paper–SPHYE-501C

(Nanophysics)

(Tuesday, 17-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—*3 Hours* *Maximum Marks*—60

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Solve any *three* of the remaining five questions (Q. No. **2** to Q. No. **6**).

(*iv*) Figures to the right indicate full marks.

1. (*a*) What is nanomaterial and nanotechnology ? 5

(*b*) Explain the magnetic properties of nanomaterial. 5

(*c*) Discuss the principle and applications of Sol-Gel Method. 5

2. (*a*) Explain one-dimensional and 2-dimensional nanostructure. 8

(*b*) Discuss microporous and mesoporous materials in detail. 7

P.T.O.

3. (*a*) Explain the magnetic properties of nanomaterials. 8

(*b*) What is surface plasmon resonance in metal nanoparticles ? Explain in detail. 7

4. (*a*) Discuss chemical bath deposition, including its mechanism and synthesis method. 8

(*b*) What is spray pyrolysis techniques ? Describe the deposition mechanisms involved in it. 7

5. (*a*) What are the principles of AFM ? Explain its working mechanism. 8

(*b*) What is the use of Fourier Transformed Infrared Spectroscopy (FTIR) and how does it work ? 7

6. (*a*) Write a short note on core shell structure. 5

(*b*) What is thin film ? Explain the use of thermal evaporation method ? 5

(*c*) What are the properties of X-ray diffraction ? 5

This question paper contains **3** printed pages]

NEPWT—02—2024

FACULTY OF SCIENCE

M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

SMATC–501

(Field Theory)

(Tuesday, 10-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 Hours *Maximum Marks*—80

N.B. :— (*i*) Question No. **1** is compulsory.

(*ii*) *All* questions carry equal marks.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Show that the polynomial $f(x) = 1 + x + ... + x^{p-1}$ is irreducible over Q, where p is prime no.

(*b*) Construct the splitting field of the polynomial $f(x) = x^4 + 1$ over Q.

(*c*) Prove that the group G (Q(α)/Q); where $\alpha^5 = 1$ and $\alpha \neq 1$, is isomorphic to the cyclic group of order 4.

P.T.O.

(*d*) Express the following symmetric polynomial as a rational function of the elementary symmetric function :

$$(x_1 - x_2)^2 (x_2 - x_3)^2 (x_3 - x_1)^2$$

2. Answer the following : 20

(*a*) State and prove Gauss lemma.

(*b*) For any field k prove that the following are equivalent :

(*i*) k is algebraically closed field

(*ii*) Every irreducible polynomial in $k[x]$ is of degree one.

3. Answer the following : 20

(*a*) Prove that any finite field F with p^n elements is the splitting field of $x^{p^n} - x \in F_p[x]$. Consequently any two finite field with p^n elements are isomorphic.

(*b*) If $f(x)$ be an irreduible polynomial over F, then prove that $f(x)$ has a multiple roots if and only if $f'(x) = 0$.

4. Answer the following : 20

(*a*) If F and E are field, then prove that distinct embedding of F into E are linearly independent over E.

(*b*) If E be a finite separable extension of a field F, then prove that the following are equivalent :

(*i*) E is a normal extension of F

(*ii*) $[E : F] = |G(E|F)|$

5. Answer the following : 20

(*a*) If a and b are constructible numbers, then $a \pm b$ are also constructible.

(*b*) Prove that regular n-gen is constructible iff Q(n) is a power of 2.

6. Answer the following : 20

(*a*) If F be a field and n be a positive integer, then prove that there exists a primitive nth root of unity in some extension E of F if and only if either char F = 0 or char F × n.

(*b*) Prove that any finite extension E of a field F is an algebraic extension.

This question paper contains **3** printed pages]

NEPWT—68—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

Paper SMATC-502

(Functional Analysis)

(Thursday, 12-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following 20

(*a*) Prove that, norm is continuous function.

(*b*) State and prove Parallelogram law.

(*c*) For an arbitrary operator, T on H, prove that, TT* & T*T are self-adjoint.

(*d*) If T is normal operator, then show that x is an eigen vector of T if and only if x is eigen vector of T*

P.T.O.

2. Answer the following : 20

(*a*) State and prove Hahn-Banach Theorem.

(*b*) State and prove the Closed Graph Theorem.

3. Answer the following : 20

(*a*) State and prove Schwartz's inequality.

(*b*) State and prove Bessel's inequality.

4. Answer the following : 20

(*a*) If T is an arbitrary operator on Hilbert space H and if α, β such that $|\alpha| = |\beta|$, then show that, $\alpha T + \beta T^*$ is normal operator on H.

(*b*) Define Unitary Operator. Prove that an operator T on Hilbert space H is unitary if and only if T is an isometric isomorphism of H onto itself.

5. Answer the following 20

(*a*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, \ldots \lambda_m$ be the distinct eigen values of T corresponding to eigen vectors $x_1, x_2, \ldots x_m$. Suppose $M_1, M_2, \ldots M_m$ are corresponding eigen spaces and $P_1, P_2, \ldots P_m$ are the projections on these eigen spaces. **If M_i's are pairwise orthogonal and spans H, then prove that, P_i's are pairwise orthogonal, $I = \sum_{i=1}^{m} P_i$ and $T = \sum_{i=1}^{m} \lambda_i P_i$.**

(*b*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, \ldots \lambda_m$ be the distinct eigen values of T corresponding to eigen vectors $x_1, x_2, \ldots x_m$. Suppose $M_1, M_2, \ldots M_m$ are corresponding eigen spaces and $P_1, P_2, \ldots P_m$ are the projections on these eigen spaces. **If T is normal operator on H, then prove that, M_i's are pairwise orthogonal.**

6. Answer the following : 20

(*a*) If N is a non-zero normed linear space, then prove that, N is Banach space if and only if $S = \{x \in N \mid \|x\| = 1\}$ is complete.

(*b*) Let T be an operator on Hilbert space H. Let $\lambda_1, \lambda_2, \ldots \lambda_m$ be the distinct eigen values of T corresponding to eigen vectors $x_1, x_2, \ldots x_m$. Suppose $M_1, M_2, \ldots M_m$ are corresponding eigen spaces and $P_1, P_2, \ldots P_m$ are the projections on these eigen spaces. **If T is normal operator on H, then prove that, each M_i reduces T.**

This question paper contains **3** printed pages]

NEPWT—134—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(NEP-2020 Pattern)

MATHEMATICS

Paper–SMATC-503

(Analytical Number Theory)

(Saturday, 14-12-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find the remainders when 2^{50} and 41^{65} are divide by 7.

(*b*) Verify that 2 is a primitive root of 19, but not of 17.

(*c*) Show that 3 is a quadratic residue of 23, but a non-residue of 31.

(*d*) Find the Bell series of Euler's totient ϕ function.

P.T.O.

2. Answer the following : 20

(a) Show that, the linear congruence $ax \equiv b$ (mod n) has a solution if and only if $d|b$, where $d = gcd\ (a, n)$. Prove that, if $d|b$, then it has d mutually incongruent solution modulo n.

(b) Solve the given system of simultaneous congruences :

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

3. Answer the following : 20

(a) If p is prime and $f(x) = a_0x^n + a_{n-1}\ x^{n-1} + \ldots\ldots\ldots\ldots + a_1x + a_0$, $a_n \not\equiv 0\ (mod\ p)$ is a polynomial of degree $n \geq 1$ with integer coefficient, then prove that the congruence $f(x) \equiv 0\ (mod\ p)$ has at most n incongruent solution modulo p.

(b) Find the order of integers 2, 3, 5 :

(i) Modulo 17

(ii) Modulo 19.

4. Answer the following : 20

(a) Let p be an odd prime and $gcd(a, p) = 1$, then prove that a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1\ (mod\ P)$.

(b) Use Gauss-Lemma to compute each of the Legendre symbols :

(i) (5/13)

(ii) (6/31).

5. Answer the following : 20

(*a*) State and prove Selberg identity

(*b*) Find the following :

(*i*) $\phi(343)$

(*ii*) $\phi(4300)$

(*iii*) $\phi(2013)$.

6. Answer the following : 20

(*a*) Calculate, $5^{110}(mod\ 131)$ and also, verify that 3 is a primitive root of 7.

(*b*) Find the values of the following Legendre symbols :

(*i*) (19/23)

(*ii*) (–23/59).

Also, define Livouville's function $\lambda(n)$ and find the table $\lambda(n)$ for $n = 1$ to 10.