This question paper contains **3** printed pages]

NEPRT—14—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

SMATC–401

(Algebra)

(Friday, 19-4-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that the multiplicative group of the nth roots of unity is a cyclic group of order n.

(*b*) Prove that an abelian group G has a composition series if and only if G is finite.

(*c*) Prove that there is no simple group of order 63.

(*d*) Prove that intersection of ideals in a ring R is again ideal.

P.T.O.

2. Answer the following : 20

(*a*) Prove that every infinite cyclic group is isomorphic to **Z**

(*b*) If N is a subgroup of G, then the following are equivalent :

(*i*) $N \Delta G$

(*ii*) $xNx^{-1} = N$ for every $x \in G$

(*iii*) $xN = Nx$ for every $x \in G$.

3. Answer the following : 20

(*a*) If G is a solvable group, then prove that every subgroup of G and every homomorphic image is solvable.

(*b*) Write down all the composition series for the Q_8.

4. Answer the following : 20

(*a*) State and prove first Sylow theorem.

(*b*) Find the non-isomorphic abelian group of order 360.

5. Answer the following : 20

(*a*) If R is a ring $a \in R$, then prove that aR is right ideal of R and Ra is left ideal of R.

(*b*) Prove that every Euclidean domain is a UFD.

6. Answer the following : 20

(*a*) If H and K are cyclic group of order m and n respectively such that $(m, n) = 1$, then prove that $H \times K$ is a cyclic group of order mn.

(*b*) Prove that a Sylow p-subgroup of a finite group G is unique if and only if it is normal.

This question paper contains **3** printed pages]

NEPRT—50—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

(SMATC-403)

(Complex Analysis)

(Wednesday, 24-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find all the values of z such that $e^z = 5 + 5i$.

(*b*) Show that the function $u(x, y) = 2xy + 3x^2y - y^3$ is harmonic. Also determine the harmonic conjugate of $u(x, y)$.

(*c*) Evaluate $\int_C 2\bar{z} - i\left(z + \frac{1}{z}\right)$, where C : $z(t) = e^{it}$ $(-\pi \leq t \leq \pi)$.

(*d*) Find the principal part of the Laurent's expansion for the function $f(z) = \frac{z}{z^2 + 4}$ valid in the neighborhood of $z = 2i$.

P.T.O.

2. Answer the following : 20

(*a*) Prove that, for given three distinct points z_1, z_2 and z_3 in extended z-plane and three distinct points w_1, w_2 and w_3 in extended w-plane there exist a unique bilinear transformation $w = T(z)$ such that $T(z_k) = w_k$ for :

$$k = 1, 2, 3.$$

(*b*) Show that the exponential function $f(z) = e^z$ is periodic function with purely imaginary period $2\pi i$. Also show the following :

(*i*) $\sin 2z = 2 \sin z \cdot \cos z$

(*ii*) $\sin\left(\frac{\pi}{2} + z\right) = \cos z.$

3. Answer the following : 20

(*a*) Define Analytic function. Prove that the necessary condition for the differentiability of the function $f(z)$ at a point $z = a$ is that $f_{\bar{z}} = 0$.

(*b*) Calculate $\int_C |z|^2 dz$ along the curves :

(*i*) $C : z_1(t) = t + it (0 \leq t \leq 1)$

(*ii*) $C : z_2(t) = t + it^2 \ (0 \leq t \leq 1).$

4. Answer the following : 20

(*a*) State and prove Taylor's theorem.

(*b*) Evaluate :

(*i*) $\int_C \frac{3z^4 + 2z - 6}{(z-2)^3} dz$

(*ii*) $\int_C \frac{e^z \sin z}{(z-2)^2} dz$

where, $C : |z| = 3.$

5. Answer the following : 20

(*a*) Define non-isolated singularity. State and prove residue theorem.

(*b*) $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx = \pi.$

6. Answer the following : 20

(*a*) Define Harmonic function. Show that the function $u(x, y) = x^3 - 3xy^2 - 2x$ is harmonic. Also find the harmonic conjugate of $u(x, y)$ and all the analytic functions $f(z)$ such that $\mathrm{Re}(f(z)) = u(x, y)$.

(*b*) Find all the singularities of the function $f(z) = \cot \pi z$. Also find the principal part of Laurent's expansion in the deleted neighborhood of the each singularity.

This question paper contains **3** printed pages]

NEPRT—82—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

(SAMATE 401-B)

(Discrete Mathematics)

(Tuesday, 30-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

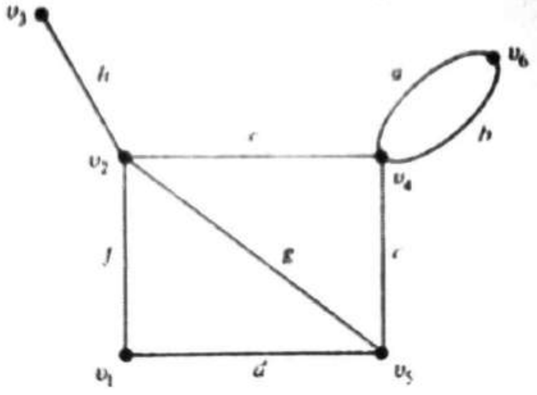
1. Answer the following : 20

(*a*) In a distributive lattice, if $b \wedge \overline{c} = 0$, then prove that $b \leq c$.

(*b*) Write a short note on isomorphism of graphs.

(*c*) Write a short note on centre of a tree with suitable examples.

(*d*) Obtain an incidence matrix of the following graph :



P.T.O.

2. Answer the following : 20

(*a*) For any a, b, c, d in lattice (A, $\leq$) , if $a \leq b$ and $c \geq d$, then prove that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$.

(*b*) For every a in a lattice (A, $\leq$), prove that :

(*i*) $a \vee a = a$ and $a \wedge a = a$

(*ii*) $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a$.

3. Answer the following : 20

(*a*) Define a simple graph. Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)/2$ number of edges.

(*b*) Define a unicursal graph with suitable examples. In a connected graph G with exactly $2k$ odd vertices, prove that there exists k edge-disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

4. Answer the following : 20

(*a*) Define a tree with suitable example. Prove that a tree with n vertices has $n - 1$ edges.

(*b*) Define that distance between two vertices and eccentricity of a vertex in a connected graph. Prove that every tree has either one or two centres.

5. Answer the following : 20

(*a*) Let A(G) be an incidence matrix of a connected graph G with n vertices. Then prove that an $(n - 1) \times (n - 1)$ submatrix of A(G) is non-singular if and only if the $n - 1$ edges corresponding to the $n - 1$ columns of this matrix constitute a spanning tree in G.

(*b*) Let B and A be, respectively, the circuit matrix and the incidence matrix of a self-loop-free graph whose columns are arranged using the same order of edges. Then prove that every row of B is orthogonal to every row A; that is $A.B^T = B.A^T = 0 \pmod 2$, where the subscript T denoted the transposed matrix.

6. Answer the following : 20

(*a*) Define Hamiltonian circuits. Prove that in a complete graph with n vertices, there are $(n - 1)/2$ edge-disjoint Hamiltonian circuits if n is an odd number ≥ 3.

(*b*) Define vertex and edge connectivity of a connnected graph with suitable examples. Prove that the vertex connectivity of graph G cannot exceed the edge connectivity of G.

This question paper contains **3** printed pages]

NEPRT—83—2024

FACULTY OF SCIENCE

M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

(SMATE-401-C)

(Dynamics and Continuum Mechanics-I)

(Tuesday, 30-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

1. Attempt the following : 5 marks each

(*a*) Write a short note on relative velocity and acceleration.

(*b*) An impulse I changes the velocity of particle of mass m from v_1 to v_2 show that kinetic energy gained is $\frac{1}{2}I(v_1 + v_2)$.

(*c*) Prove that "total external impulse applied to the system of particle is equal to the total change of linear momentum".

(*d*) Prove that central force of type $F = F(r)\bar{r}$ is conservative.

P.T.O.

2. Attempt the following : 10 marks each

(*a*) Derive an expression for vector moment about a point and scalar moment about an axis.

(*b*) A particle is constrained to move along equiangular spiral so that the radius vector moves with constant angular velocity. Determine the velocity and acceleration components.

3. Attempt the following : 10 each

(*a*) Prove that, "Earth's gravitational force is conservative".

(*b*) A uniform rectangular lamina ABCD is such that AB = $2a$, BC = $2b$. Find the direction of the principal axes at A.

4. Attempt the following : 10 marks each

(*a*) Prove that, "The K.E. of a system of particle moving generally in a space is equal to the some of K.E. of a single particle of total mass equal to that of the system, concentrated at its centroid and moving with centroid's velocity, together with the K.E. of the system in its.

(*b*) Particle A, B, C of masses m_1, m_2, m_3 lie on a smooth horizontal table and are connected together by taut inextensible strings AB, BC. The angle ABC is $\pi - \alpha, \alpha$ being acute. If an impulse I is applied to C along $\overline{\text{BC}}$ show that B starts to move indirection making with $\overline{\text{AB}}$ an angle $\tan^{-1}\left[\left(1+\frac{m_1}{m_2}\right)\tan\alpha\right]$, find initial velocity of A.

5. Attempt the following : 10 marks each

(*a*) Show that the products in inertia with respect to the principal axes are zero.

(*b*) A uniform rigid rod AB moves so that A and B have velocities u_A, u_B at any instant.

Show that the K.E. is then $T = \frac{1}{6}M(u_A^2 + u_A u_B + u_B^2)$, M being the mass.

6. Attempt the following : 10 marks each

(*a*) Prove that for single particle in conservative field of force the sum of kinetic and potential energies is constant.

(*b*) Find an equimomental system of particles for a uniform rod AB of mass M, where O be the centroid of the rod, $2a$ its length ?

This question paper contains **3** printed pages]

NEPRT—81—2024

FACULTY OF SCIENCE/ARTS

M.Sc./M.A. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

(SMATE-401-A)

(Ordinary Differential Equations)

(Tuesday, 30-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Solve :

$$y'' + 16y = 0.$$

(*b*) Verify that $\phi_1(x) = e^x (x > 0)$ satisfies the equation $xy'' - (x + 1) y' + y = 0$ and find the second independent solution.

(*c*) Solve :

$$x^3y''' + 2x^2y'' - xy' + y = 0 \text{ for } x > 0.$$

(*d*) Solve :

$$2xydx + (x^2 + 3y^2)\, dy = 0.$$

P.T.O.

2. Answer the following : 20

(*a*) Prove that every solution ψ of $L(y) = b(x)$ on I is $\psi = \psi_p + c_1\phi_1 + c_2\phi_2$, where ψ_p is particular solution. Where ϕ_1, ϕ_2 are two linearly independent solutions of $L(y) = 0$ and c_1, c_2 are constants.

(*b*) Solve :

$$y'' - y' - 2y = e^{-x}.$$

3. Answer the following : 20

(*a*) One solution of $L(y) = 0$ on interval I is v_2 .... v_n is any basis on I for the solutions of the linear equation $\phi_1 v^{(n-1)} + \ldots\ldots +$ $\left[n\phi_1^{(n-1)} + a_1(n-1)\phi_1^{(n-2)} + \ldots\ldots + a_{n-1}\phi_1\right] v = 0$ of order $n - 1$ and if $v_k = u_k$, $(k = 2, 3, \ldots\ldots, n)$, then prove that $\phi_1, u_2\phi_1, \ldots\ldots, u_n\phi_1$ is a basis for the solution of $L(y) = 0$ on I.

(*b*) Solve $y'' - xy = 0$ by using power series expansion.

4. Answer the following : 20

(*a*) Prove that the solution of $x^2y'' + a(x)xy' + b(x)y = 0$ is given by $\phi_i(x) = |x|ri \sum_{k=0}^{\infty} C_k x^k$, where $i = 1, 2$ and a, b have convergent power series expansions.

(*b*) Find two solution of Bessel equation.

5. Answer the following : 20

(*a*) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I if and only if it is solution of the intergral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$ on I.

(*b*) Prove that the *k*th successive appoximation ϕ_k to the solution ϕ of the inital value problem $y' = f(x, y)$, $y(x_0) = y_0$ satisfies $|\phi(x) - \phi_k(x)| \leq \frac{M\,(K\alpha)^{k+1}}{K(k+1)\,!}e^{k\alpha}$ for all x in I, where k is Lipschitz constant and $|f(x, y)| \leq M$.

6. Answer the following : 20

(*a*) Solve :

$$y^{(3)} - 4y' = 0.$$

(*b*) Solve :

$$y' = xy,\ y(0) = 1.$$

This question paper contains 4 printed pages]

NEPRT—32—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A/M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

SMARTC–402

(Real Analysis)

(Monday, 22-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove that $fg \in \mathbf{R}(\alpha)$.

(*b*) Let $f_n(x) = \dfrac{\sin x}{\sqrt{n}}$, ($x \in R$, $n = 1, 2, \ldots\ldots\ldots\ldots$), then show that $\lim_{n\to\infty} f_n'(0) \neq f'(0)$.

(*c*) Does the every member of an equicontinuous family is uniformly continuous ? Justify your answer.

(*d*) Let $f : R^2 \to R^2$ be defined by the equation $f(x, y) = (e^x \cos y, e^x \sin y)$. Calculate Df and $|Df|$.

P.T.O.

2. Answer the following : 20

(*a*) If $f_1 \in \mathbf{R}(\alpha)$ and $f_2 \in R(\alpha)$ on $[a, b]$, then prove that $f_1 + f_2 \in \mathbf{R}(\alpha)$ and $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.

(*b*) Suppose $f \geq 0$, f is continuous on $[a, b]$ and $\int_a^b f(x)dx = 0$. Show that $f(x) = 0$ for all $x \in [a, b]$. Also, show that, if f is continuous on $[a, b]$, then $f \in \mathbf{R}(\alpha)$ on $[a, b]$

3. Answer the following : 20

(*a*) State and prove the Cauchy criterion for uniform convergence.

(*b*) (*i*) For $m = 1, 2, 3, \ldots\ldots, n = 1, 2, \ldots\ldots$, let $S_{mn} = \dfrac{m}{m+n}$

Show that :

$$\lim_{n\to\infty} \lim_{m\to\infty} S_{mn} \neq \lim_{m\to\infty} \lim_{n\to\infty} S_{mn}.$$

(*ii*) Let $f_n(x) = \dfrac{x^2}{(1+x^2)^n}$, $(x \in R, n = 0, 1, 2, \ldots\ldots)$. Then prove that the series $\sum_{n=0}^{\infty} f_n(x)$ of continuous functions converges to a discontinuous sum.

4. Answer the following : 20

(*a*) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$ $(|x| < R)$.

Then $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on [–R + ε, R – ε], no matter which ε > 0 is chosen. the function f is continuous and differentiable on (–R, R) and $f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1}$ ($|x| < R$).

(*b*) If f is continuous on [0, 1] and if $\int_0^1 f(x)x^n dx = 0$, $n = 0, 1, 2, \ldots$, show that $f(x) = 0$ on [0, 1].

5. Answer the following : 20

(*a*) (*i*) Let $A \subseteq R^m$, let $f : A \to R^n$, if f is differentiable at $\bar{a}$, then prove that all the directional derivative of f at $\bar{a}$ exist and $f' (\bar{a}, \bar{u}) = Df(\bar{a}).\, \bar{u}$

(*ii*) Let $A \subseteq R^m$, let $f : A \to R^n$, if f is differentiable at $\bar{a}$, then prove that f is continuous at $\bar{a}$.

(*b*) Let $f : R^2 \to R^3$ and $g : R^3 \to R^2$ be given by the equation :

$f(\bar{x}) = \left(e^{2x_1 + x_2}, 3x_2 - \cos x_1, x_1^2 + x_2 + 2\right)$

$g(\bar{x}) = \left(3y_1 + 2y_2 + y_3^2, y_1^2 - y_3 + 2\right)$

(*i*) If $G(\bar{y}) = f\left(g(\bar{y})\right)$, find $DG(\bar{0})$.

(*ii*) If $f(\bar{x}) = g\left(f(\bar{x})\right)$, find $DF(\bar{0})$.

6. Answer the following : 20

(*a*) (*i*) Suppose F and G are differentiable functions on [a, b], F' = $f \in$ **R** and G' = $g \in$ **R**, then show that

$$\int_a^b f(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx.$$

(*ii*) Let $f_n(x) = n^2 x\ (1 - x^2)^n$, $0 \le x \le 1$ and $n = 1, 2, 3, \ldots\ldots$, show that :

$$\lim_{x \to \infty} \left[\int_0^1 f_n(x) dx \right] \neq \int_0^1 \left[\lim_{n \to \infty} f_n(x) \right] dx.$$

(*b*) State the Stone-Weierstrass theorem.

Define $f : \mathrm{R}^2 \to \mathrm{R}$ by setting $f(\overline{0}) = \overline{0}$ and $f(x, y) = \dfrac{x^2}{x^4 + y^2}$, if $f(x, y) \neq \overline{0}$.

Show that all directional derivatives of f exist to $\overline{0}$ but that f is not differentiable at $\overline{0}$.

This question paper contains 4 printed pages]

NEPRT—190—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (NEP) (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper (SMATE-452)

(Combinatorics)

(Monday, 29-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) How many different sequences of heads nd tails are possible if a coin is flipped 100 times ? Using the fact that $2^{10} = 1024 \approx 1000 = 10^3$, give your answer in terms of an approximate power of 10.

(*b*) Find the generating function for a_r, the number of ways to select r balls from three green, three white and three gold balls.

(*c*) Find a recurrence relation for number of ways to arrange n distinct objects on a row. Find the number of arrangements of eight objects.

P.T.O.

(*d*) If a school has 100 students with 50 students taking French, 40 students taking Latin, and 20 students taking both languages how many students take no language ?

2. Answser the following : 20

(*a*) Prove that the number of selections with repetition of r objects chosen from n type of objects is given by $C(r + n - 1, r)$.

(*b*) Given five distinct pairs of gloves, 10 distinct gloves in all, how many ways are there to distribute two gloves to each of five sisters :

(*i*) If the two gloves someone recieves might both be for the left hand or right hand ?

(*ii*) If each sister gets one left-hand glove and one right hand glove ?

3. Answer the following : 20

(*a*) Use a generating function to model the problem of counting all selections of six objects chosen from three types of objects with repetition of up to four objects of each type. Also model the problem with unlimited repetition.

(*b*) Use generating function to find the number of ways to collect Rs. 15 and 20 distinct people if each of the first 19 people can give a rupee (or nothing) and the twentieth person can give either Rs. 1 or Rs. 5 (or nothing).

4. Answer the following : 20

(*a*) Find recurrence relations for :

(*i*) The number of n-digit ternary sequences with an even number of 0s.

(*ii*) The number of n-digit ternary sequences with an even number of 0s and an even number of 1s.

(*b*) Find a recurrence relation for the number of ways to distribute n distinct objects onto four boxes. What is the initial condition ?

5. Answer the following : 20

(*a*) Let $A_1, A_2, \ldots\ldots\ldots, A_n$ be n sets in the universal set U of N elements. Let S_k denote the sum of the sizes of all k-tuple intersection of the $A_i s$. Then prove that :

$$N(\overline{A}_1 . \overline{A}_2 \ldots . \overline{A}_n) = N - S_1 + S_2 - S_3 + \ldots . + (-1)^k S_k + \ldots\ldots + (-1)^n S_n.$$

(*b*) How many ways are there to send six different birthday cards, denoted C_1, C_2, C_3, C_4, C_5, C_6, to three aunts and three uncles, denoted A_1, A_2, A_3, U_1, U_2, U_3, if the aunt A_1 would not like cards C_2 and C_4; if A_2 would not like C_1 or C_5; if A_3 likes all cards; if U_1 would not like C_1 or C_5; if U_2 would not like C_4; and if U_3 would not like C_6 ?

6. Answer the following : 20

(*a*) How many ways are there to distribute 36 identical jelly beans among four children :

(*i*) Without repetition ?

P.T.O.

(*ii*) With each child getting 9 beans ?

(*iii*) With each child getting at least 1 bean ?

(*b*) A bank pays 4 percent interest each year on maney on savings account. Find recurrence relations for the amount of money a gnome would have after n years if it follows the investment strategies of :

(*i*) Investing Rs. 1000 and leaving it in the bank for n years.

(*ii*) Investing Rs. 100 at the end of each year.

This question paper contains 4 printed pages]

NEPRT—191—2024

FACULTY OF SCIENCE & TECHNOLOGY

M.A/M.Sc. (Second Semester) (NEP) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

SMATE–453

(Dynamics and Continuum Mechanics-II)

(Monday, 29-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If $\phi = x_1 x_2 + 2x_3$, find a unit vector $\bar{n}$ normal to the surface of constant passing through point (2, 1, 0)

(*b*) Given $T_{ij} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

decompose the tensor into synmmetric and an antisymmetric part.

P.T.O.

(*c*) Show that there is no shearing stress on any plane containing the point.

If the state of stress at a certain point is $\bar{T} = -P\bar{I}$ where P is scalar.

(*d*) Discuss material derivative.

2. Answer the following : 20

(*a*) Write a short note on Dyadic product of vector

(*b*) Find the eigen values and eigen vector for tensor $[T] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$

3. Answer the following : 20

(*a*) Consider the Scalar field

$\phi = x_1^2 + 3x_1x_2 + 2x_3$:

(*i*) Find unit vector normal to the surface of derivative of ϕ at origin and at (1, 0, 1)

(*ii*) What is max value of the directional derivative of ϕ at origin and at (1, 0, 1).

(*iii*) Evaluate $\frac{d\phi}{dr}$ at the origin if $dr = ds\ (e_1 + e_3)$.

(*b*) Discuss divergence of a vector field, divergence to tensor field and curl of vector field.

4. Answer the following : 20

(*a*) If $d\overline{x} = (ds)\hat{n}$, where $\hat{n}$ is a unit vector in the direction of $d\overline{x}$, then show that, $\frac{1}{ds}\frac{\mathrm{D}}{\mathrm{D}t}(ds) = \hat{n}\,D\,\hat{n} = Dn \times n$.

(*b*) Given the motion of a continuum to be :

$x_1 = \mathrm{X}_1 + \mathrm{Kt}\mathrm{X}_2,\ x_2 = (1 + kt)\ \mathrm{X}_2,\ x_3 = \mathrm{X}_3.$

If the temperature field is given by the spatial description $\Theta = \alpha(x_1 + x_2)$, then find $\frac{\mathrm{D}\Theta}{Dt}$.

5. Answer the following : 20

(*a*) Define stress tensor and show that,

$\mathrm{T}\ \left(n_1\overline{e}_1 + n_2\overline{e}_2 + n_3\,\overline{e}_3\right) = n_1\mathrm{T}\ (e_1) + n_2\mathrm{T}\ (e_2) + n_3\ \mathrm{T}(e_3).$

(*b*) The distribution of stress inside a body is given by the matrix :

$$[\mathrm{T}] = \begin{bmatrix} -p+\rho gy & 0 & 0 \\ 0 & -p+\rho gy & 0 \\ 0 & 0 & -p+\rho gy \end{bmatrix}$$

where p, ρ, g are constants :

(*i*) What is distribution of the stress vector on the six faces of block ?

(*ii*) Find the total resultant force acting on the face $y = 0$ and $x = 0$.

6. Answer the following : 20

(*a*) Given that R is a rotation tensor and that m is a unit vector in the direction of the axis of rotation, prove that the dual vector q of R^A is parallel to m.

(*b*) Define strain tensor. Write its components.

This question paper contains **4** printed pages]

NEPRT—110—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (NEP) (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

MATHEMATICS

SMATC–451

(Linear Algebra)

(Thursday, 18-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following (**5** marks each) :

(*a*) Let V be a vector space over field F, and $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly independent, then prove that S_1 is also linearly independent.

(*b*) If V and W are vector spaces over field F and $T : V \to W$ be a linear and invertible, then prove that $T^{-1} : W \to V$ is linear.

P.T.O.

(*c*) If $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$, then find all eigen values and corresponding eigen vectors of A.

(*d*) For $x = (a_1, a_2, ... a_n)$, $y = (b_1 .... b_n) \in F^n$ and (,) : $F^n \times F^n \to F$ be a function defined by :

$$(x, y) = \sum_{i=1}^{n} a_i \overline{b}_i .$$

Then prove that (x, y) be an inner product on $\mathbf{F}^n$.

2. Answer the following (**10** marks each) :

(*a*) If S be a linearly independent subset of a vector space, V and $v \in V$ be a vector such that $v \notin S$. Then prove that SU$\{v\}$ is linearly dependent iff $V \in$ Span(S).

(*b*) If W be a subspace of a finite dimensional vector space V. Then prove that W is also finite dimensional and dim (W) $\leq$ dim(V).

3. Answer the following (**10** marks each) :

(*a*) Let V and W are vector space and $T : V \to W$ be a linear transformation. If V is a finite dimensional, then prove that :

$$\dim(V) = \text{rank } (T) + \text{nullity } (T).$$

(*b*) If V and W are vector space and $T : V \to W$ be a linear mapping, then prove that T is one-one iff $N(T) = \{0\}$.

4. Answer the following (**10** marks each) :

(*a*) Let A be an $m \times n$ matrix. If P and Q are $m \times m$ and $n \times n$ matrices respectively, then prove that :

(*i*) rank (AQ) = rank (A)

(*ii*) rank (PA) = rank (A).

(*b*) Let AX = B be a system of linear equations. Then prove that system AX = B is consistent iff rank(A) = rank (A|B).

5. Answer the following (**10** marks each) :

(*a*) If T and U are linear operators on inner product space V over F, then prove that :

(*i*) $T^{**} = T$, (*ii*) $I^{*} = I$.

(*b*) Let V be an inner product space and S = $\{V_1, V_2, ... V_k\}$ be an orthogonal subset of V consisting non-zero vectors. If $y \in$ span(S), then prove that :

$$y = \sum_{i=1}^{k} \frac{(y, V_i)}{\|V_i\|^2} V_i \quad .$$

P.T.O.

6. Answer the following (**10** marks each) :

(*a*) If T, U_1 and U_2 are linear operator on vector space V, then prove that :

(*i*) $T(U_1 + U_2) = TU_1 + TU_2$

(*ii*) $T(U_1U_2) = (TU_1)\,U_2.$

(*b*) State and prove parallelogram law on an inner product space V.

This question paper contains **3** printed pages]

NEPRT—131—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper SMATC-452

(Measure and Integration Theory)

(Saturday, 20-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

Note :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Show that, outer measure is translation invariant.

(*b*) Give an example to show that :

$$D^+(f + g) \neq D^+ f + D^+ g$$

P.T.O.

(*c*) Let $f = g$ *a. e* (μ), where μ is a complete measure. Show that if f is measurable, so is g.

(*d*) Show that if v_1, v_2 and μ are measures and :

$$v_1 \perp \mu, v_2 \perp \mu, \text{ then } v_1 + v_2 \perp \mu.$$

2. Answer the following : 20

(*a*) Prove that, the outer measure of an interval equals its length.

(*b*) (*i*) Show that, if $m^*(A) = 0$, then prove that :

$$m^*(A \cup B) = m^*(B),$$

for any set B.

(*ii*) Show that the set [0, 1] is uncountable.

3. Answer the following : 20

(*a*) If $f \in L(a, b)$ then prove that $\int_a^x f(t)dt$ is a continuous function on $[a, b]$ and :

$$f \in BV[a, b]$$

(*b*) Let f be defined by :

$$f(x) = x \sin(1/x) \text{ for } x \neq 0, f(0) = 0,$$

find the four derivatives at $x = 0$.

4. Answer the following : 20

(*a*) Define Hereditary. Let μ^* be an outer measure on H(R) defined by μ on R, then show that S* contains S(R), the σ-ring generated by R.

(*b*) Show that :

$$H(R) = \{E ! E \subseteq U_{n=1}^{\infty} E_n E_n \in R\}$$

5. Answer the following : 20

(*a*) State and prove Hahn decomposition theorem.

(*b*) Show that if :

$$\phi(E) = \int_E f d\mu$$

where $\int f d\mu$ is defined, then ϕ is a signed measure.

6. Answer the following : 20

(*a*) Show that the continuous functions are measurable. Also show that BV[*a*, *b*] is a vector space over the real numbers.

(*b*) (1) Show that, if μ is a σ-finite measure on R, then the extension $\bar{\mu}$ of μ to s* is also σ-finite.

(2) State Radon-Nikodym theorem.

This question paper contains **3** printed pages]

NEPRT—189—2024

FACULTY OF SCIENCE / ARTS

M.Sc. (NEP) (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper SMATE-451(A)

(Partial Differential Equations)

(Monday, 29-4-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Solve :

$$yzp + xzq = x + y.$$

(*b*) Prove that there always exists an integrating factor for Pfaffian differential equation in two variables.

P.T.O.

(*c*) Solve :

$$u_{xx} - u_{tt} = 0\,.$$

(*d*) Show that the solution of the Dirichlet problem if it exists is unique.

2. Answer the following : 20

(*a*) Let $u(x, y)$ and $v(x, y)$ be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u, v)}{\partial(x, y)} = 0$, then prove that there exist a relation between u and v not involving x and y explicity.

(*b*) If $\bar{X}.\text{curl}\,\bar{X} = 0$, where $X = P\bar{i} + Q\bar{j} + R\bar{k}$ and μ is an arbitrary differentiable function of x, y and z, then prove that $\mu\bar{X}.\text{curl}(\mu\bar{X}) = 0$.

3. Answer the following : 20

(*a*) Find complete integral of $2(z + xp + yq) = yp^2$ by Charpit's method.

(*b*) Find the integral surface of the partial differential equation $p^2x + qy - z = 0$ containing the initial line $x_0(s) = s$, $y_0(s) = 1$, $z_0(s) = -s$.

4. Answer the following : 20

(*a*) Reduce into canonical forms and solve $u_{xx} - (x^2)u_{yy} = 0$.

(*b*) Find an expression of d' Alembert's solution which describes the vibrations of an infinite string.

5. Answer the following : 20

(*a*) Explain analytic expression for the Monge cone at (x_0, y_0, z_0).

(*b*) Show that the solution for the Dirichlet problem for a circle is given by the Poisson integral formula.

6. Answer the following : 20

(*a*) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x), x \geq 0$.

(*b*) Find complete integral of $p^2 + q^2 = x + y$.

This question paper contains **2** printed pages]

NEPRT—152—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper SMATC-453

(Topology)

(Tuesday, 23-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* questions from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) If X is a non-empty set, then show that τ_d = {set of all subjects of X} is a topology on X.

(*b*) If A is subspace of X, then show that the inclusion mapping $j : A \rightarrow X$ is continuous function.

(*c*) If X is finite set, then show that a discrete topological space is compact.

(*d*) Show that the set of real numbers with usual topology is normal space.

2. Answer the following : 20

(*a*) If Y be the subspace of X, then prove that a set A is closed in Y if and only if A equals the intersection of closed set of X with Y.

P.T.O.

(*b*) Define basis of the topology. If B′ denotes the collection of all rectangular regions whose sides are parallel to the coordinate axes. B denotes the collection of all circular regions, then show that topologies generated by B and B′ are same.

3. Answer the following : 20

(*a*) If X and Y are topological spaces then prove that following conditions are equivalent :

(*i*) $f : X \to Y$ is continuous.

(*ii*) If V is closed in Y, then $f^{-1}(V)$ is closed in X.

(*b*) Prove that the product of connected set is a connected set.

4. Answer the following : 20

(*a*) If X is compact and Y is Hausdorff space and $f : X \to Y$ is bijective continuous function, then prove that f is homeomorphism.

(*b*) Define locally compact topological space. Prove that if X is compact topological space, then X is locally compact topological space.

5. Answer the following : 20

(*a*) Prove that closed subspace of normal space is normal.

(*b*) Let (X, τ) be a topological space, then prove the following :

(*i*) Subspace of first countable space is first countable space.

(*ii*) Subspace of second countable space is second countable space.

6. Answer the following : 20

(*a*) If (X, τ) be a topological space, $A \subseteq X$, then prove that $\bar{A} = A \cup A'$.

(*b*) Prove that subspace of Regular space is regular space.

This question paper contains **3** printed pages]

RT—272—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper IV

(Complex Analysis-I)

(Wednesday, 24-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following :

(*a*) Prove that, the mapping $w = \frac{1}{z}$, $z \neq 0$ maps the circles and straight lines onto circles and straight lines. 15

Or

(*b*) (*i*) Find a bilinear transformation mapping the points $z_1 = i$, $z_2 = 2$, $z_3 = -2$ onto the points $w_1 = i$, $w_2 = 1$, $w_3 = -1$.

(*ii*) Find the image of half-plane $\mathrm{Re}(z) > 0$ under the transformation : 15

(1) $w = 2iz - i$

(2) $w = \frac{i}{z} - 1$.

P.T.O.

2. Attempt the following :

(*a*) Define Branch of logarithm and principle branch of logarithm. If $z = x + iy$ and $z \neq 0$. Find all the values of log z. For $z \neq 0$, prove that $\log |z| \leq |\log z| \leq \log |z| + |\text{Arg } z|$. 15

Or

(*b*) Find all the values of z for which :

(*i*) $e^{3z} = 1$

(*ii*) $e^{e^{z}} = 1$

(*iii*) $e^{z^2} = 1$

(*iv*) $\sec^{-1} i$

(*v*) 2^{i^i}

3. Attempt the following :

(*a*) Define Analytic function. If $f(z) = u(x, y) + iv(x, y)$ is defined in domain D and $u(x, y)$ and $v(x, y)$ are continuous with continuous partials that satisfy the Cauchy-Reimann equations $u_x = v_y$ and $u_y = -v_x$ in D, then prove that $f(z)$ is analytic in D. 15

Or

(*b*) Define Harmonic function. 15

Show that the function $u(x, y) = e^x(x \cdot \cos y - y \cdot \sin y)$ is harmonic. Also find the harmonic conjugate of $u(x, y)$ and all the analytic functions $f(z)$ such that $\text{Re}(f(z)) = u(x, y)$.

4. Attempt the following :

(*a*) State and prove cauchy's main theorem. 15

Or

(*b*) Calculate $\int_C z^2 \, dz$ along the curves :

(*i*) $C : z_1(t) = t + it \ (0 \leq t \leq 1)$

(*ii*) $C : z_2(t) = t + it^2 \ (0 \leq t \leq 1)$.

5. Attempt any *three* of the following : 15

(*a*) Find all the values of log $(1 + i)$ and log 1.

(*b*) Find the values of a, b and c such that the function $f(z) = a(x^2 + y^2) + ibxy + c$ is an entire function.

(*c*) Find all the values of :

(*i*) $\sec^{-1} i$

(*ii*) $\cos^{-1} \frac{\sqrt{2}}{2}$.

(*d*) Find the lenght of the curve :

$$C : z(t) = 3e^{2it} + 2 \ (-\pi \leq t \leq \pi).$$

This question paper contains 4 printed pages]

RT—367—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper-V

(Discrete Mathematics)

(Tuesday, 30-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) Define the universal upper and lower bounds in a lattice with a suitable example. Let (A, $\leq$) be a lattice with universal upper and lower bounds 1 and 0. Then for every a in (A, $\leq$), prove that :

$$a \vee 1 = 1 \qquad a \wedge 1 = a$$

$$a \vee 0 = a \qquad a \wedge 0 = 0.$$

Or

(*b*) Define a boolean algebra with a suitable example. With usual notations, for any a and b in boolean algebra, prove that :

$$\overline{a \vee b} = \bar{a} \wedge \bar{b} \text{ and } \overline{a \wedge b} = \bar{a} \vee \bar{b}.$$

P.T.O.

2. Attempt the following : 15

(*a*) Define a connected and a disconnected graph with a suitable example. Prove that a graph G is disconnected if and only if it vertex set V can be partitioned into two non-empty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in the subset V_1 and the other in subset V_2.

Also prove that if a graph (connected or disconnected) has exactly two vertices of odd degree, then there must be a path joining these two vertices.

Or

(*b*) Define degree of a vertex in a graph. Prove that the number of vertices of odd degree in a graph is always even.

In a connected graph G with exactly $2k$ odd vertices, prove that there exists k edge-disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

3. Attempt the following : 15

(*a*) Define the distance between two vertices and eccentricity of a vertex in a connected graph. Prove that every tree has either one or two centres.

Prove that a connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one circuit.

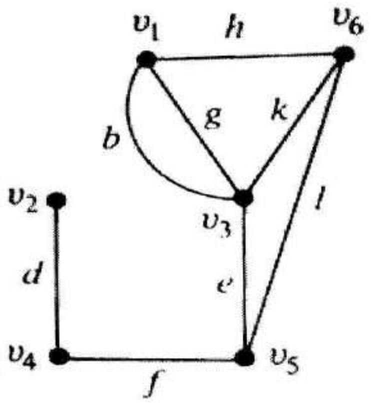
Or

(*b*) Define vertex and edge connectivity of a graph with suitable examples. Prove that the vertex connectivity of a graph G cannot exceed the edge connectivity of G.

4. Attempt the following : 15

(*a*) If A(G) is an incidence matrix of a connected graph G with n vertices, then prove that the rank of A(G) is $n - 1$.

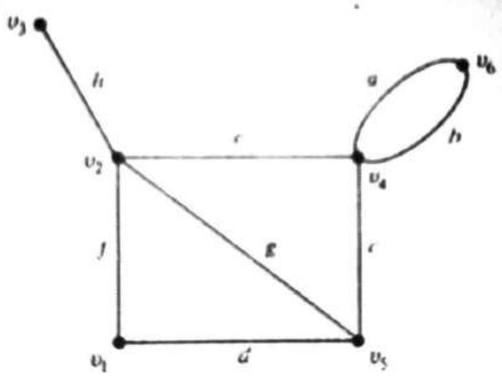
Also, obtain the incidence matrix of the following graph.



Or

(*b*) If B is a circuit matrix of a connected graph G with e edges and n vertices, then prove that rank of B = $e - n + 1$.

Obtain the circuit matrix of the following graph.



P.T.O.

5. Attempt any *three* of the following : 15

(*a*) Let a, b, c be elements in a lattice (A, $\leq$). If $a \leq b$, then show that $a \vee (b \wedge c) \leq b \wedge (a \vee c)$.

(*b*) Write a short note on operation on a graph.

(*c*) Prove that every connected graph has at least one spanning tree.

(*d*) Write a short note on directed graph.

This question paper contains **2** printed pages]

RT—15—2024

FACULTY OF SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–I

(Abstract Algebra) (Group & Ring Theory)

(Tuesday, 16-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

   (*a*) State and prove first Isomorphism Theorem of a group.

   (*b*) Prove that every cyclic group of order n is isomorphic to Z_n further more, prove that multiplicative group of nth root of unity is a cyclic group of order n.

2. Attempt any *one* of the following : 15

   (*a*) If order of group G is p^n, then prove that G has non-trivial center. Also prove that every group of p^2 is abelian.

   (*b*) Prove that every subgroup of solvable group is solvable.

P.T.O.

3. Attempt any *one* of the following : 15

(*a*) Prove that every finitely generated abelian group G is a direct product of finite number of cyclic group.

(*b*) State and prove finitely generated abelian group.

4. Attempt any *one* of the following : 15

(*a*) Prove that every Euclidian domain is PID.

(*b*) If $f : \mathrm{R} \rightarrow \mathrm{S}$ be a homomorphism of a ring R into a ring S, then prove that $\ker(f) = \{0\}$ iff f is one-one.

5. Attempt any *three* of the following : 15

(*a*) Prove that sum of two ideal is ideal.

(*b*) Find all non-isomorphic abelian group of order 81

(*c*) Find all normal series and composition series of $\mathbf{Z}_6$.

(*d*) Prove that Intersection of two ideal in a ring R is again ideal.

[This question paper contains **1** printed page]

RT-176-2024

FACULTY OF SCIENCE

M.Sc. Mathematics (First Year) (Semester-I)

April , 2024

Paper No.: III

(Ordinary Differential Equations P-III (New CBCS Pattern)

(Monday, 22-04-2024) **Time: 10.00 a.m. to 01.00 p.m.**

Time - Three Hours **Maximum Marks-75**

N.B: 1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q,No.1) Attempt any one of the following **(15 marks)**

A) Prove that the two solutions $\phi_1(x), \phi_2(x)$ of L(y) = 0 are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I

B) Prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$, Where ϕ_1 , ϕ_2 are two linearly independent solutions of L(y) = 0 on an interval I containing a point x_0 .

Q,No. 2) Attempt any one of the following **(15 marks)**

A) Solve $y'' - xy = 0$ by using power series expansion.

B) Derive two solutions of Legendre equation

Q,No. 3) Attempt any one of the following **(15 marks)**

A) Find two solution of Bessel Equation.

B) Derive two solutions of Legendre equation

Q,No. 4) Attempt any one of the following **(15 marks)**

A) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I.

B) Prove that $M(x, y) + N(x, y)y' = 0$ is exact on some rectangle R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. And solve $2xydx + (x^2 + 3y^2)dy = 0$

Q, No. 5) Attempt any three of the following **(15 marks)**

i) Solve $y^{(3)} - 4y' = 0$

ii) Find all real valued solutions of $y' = \frac{e^{x-y}}{1+e^x}$

iii) Solve $y'' + 16y' = \cos x$

iv) Verify that $\phi_1(x) = x (x > 0)$ satisfies the equation $x^2 y'' - xy' + y = 0$ and find the second independent solution.

This question paper contains **4** printed pages]

RT—95—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper II(A)

(Real Analysis)

(Friday, 19-4-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) Prove that $f \in \mathbf{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition p on $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$. Also, suppose $f \geq 0$, f is continuous on $[a, b]$ and $\int_a^b f(x)\, dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

Or

(*b*) If $f_1 \in \mathbf{R}(\alpha)$ and $f_2 \in \mathbf{R}(\alpha)$ on $[a, b]$, then show that $f_1 + f_2 \in \mathbf{R}(\alpha)$ and

$$\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha.$$

P.T.O.

2. Attempt the following : 15

(*a*) State and prove the Cauchy criterion for uniform convergence. Furthermore, let $f_n(x) = \frac{\sin x}{\sqrt{n}}, (x \in \mathbf{R}, n = 1, 2, ...)$, then prove that :

$$\lim_{n\to\infty} f_n'(0) \neq f'(0).$$

Or

(*b*) Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathbf{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, ...$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathbf{R}(\alpha)$ on $[a, b]$ and

$$\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha.$$

3. Attempt the following : 15

(*a*) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < \mathrm{R}$, and define :

$$f(x) = \sum_{n=0}^{\infty} c_n x^n \quad (|x| < \mathrm{R}).$$

Then $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-\mathrm{R} + \varepsilon, \mathrm{R} - \varepsilon]$, no matter which $\varepsilon > 0$ is chosen. The function f is continuous and differentiable on $(-\mathrm{R}, \mathrm{R})$, and :

$$f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1} \quad (|x| < \mathrm{R}).$$

Or

(*b*) Given a double sequence $\{a_{ij}\}. i = 1, 2, 3 \ldots\ldots, j = 1, 2, 3 \ldots\ldots,$ suppose that :

$$\sum_{j=1}^{\infty} |a_{ij}| = b_i \quad (i = 1, 2, 3, \ldots\ldots),$$

and $\sum b_i$ converges. Then prove that :

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}.$$

4. Attempt the following : 15

(*a*) (I) Let $A \subseteq R^m$, let $f : A \to R^n$, if f is differentiable at $\bar{a}$, then prove that all the directional derivative of f at $\bar{a}$ exist and $f'(\bar{a}, \bar{u}) = Df(\bar{a}).\bar{u}$.

(II) Let $A \subseteq R^m$, let $f : A \to R^n$, if f is differentiable at $\bar{a}$, then prove that f is continuous at $\bar{a}$.

Or

(*b*) State implicit function theorem. Given $f : R^5 \to R^2$ of class C^1. Let $\bar{a} = (1, 2, -1, +3, 0)$. Suppose that $f(\bar{a}) = \bar{0}$ and

$$Df(\bar{a}) = \begin{bmatrix} 1 & 3 & 1 & -1 & 2 \\ 0 & 0 & 1 & 2 & -4 \end{bmatrix}.$$

(I) Show there is a function $f : B \to R^2$ of class C^1 defined on an open set B of R^3 such that : $f(x_1, g_1(x), g_2(x), x_2 x_3) = \bar{0}$ for $\bar{x} = (x_1, x_2, x_3) \in B$ and $g = (1, 3, 0) = (2, -1)$

(II) Find $Dg(1, 3, 0)$.

5. Attempt any *three* of the following : 15

(*a*) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$, then prove that $fg \in \mathbf{R}(\alpha)$.

(*b*) For $m = 1, 2, 3, \ldots\ldots$, $n = 1, 2, \ldots\ldots\ldots$, let

$$S_{mn} = \frac{m}{m+n},$$

Show that :

$$\lim_{n\to\infty} \lim_{m\to\infty} S_{mn} \neq \lim_{m\to\infty} \lim_{n\to\infty} S_{mn}.$$

(*c*) Prove that every uniformly convergent sequence of bounded function is uniformly bounded.

(*d*) Define :

$$f : R^2 \to R \text{ by setting } f(\bar{0}) = \bar{0} \text{ and}$$

$$f(x, y) = \frac{x^2}{x^4 + y^2}, \text{ if } f(x, y) \neq \bar{0}.$$

Show that all directional derivative of f exist to $\bar{0}$ but that f is not differentiable at $\bar{0}$.

This question paper contains 4 printed pages]

NEPRT—84—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

(SAMATE-401-D)

(Theory of Probability)

(Tuesday, 30-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Attempt any *three* questions from Q. Nos. **2** to **6**.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable calculator is allowed.

(*vi*) Use of table for area under standard normal curve is allowed.

1. Answer the following : 20

(*a*) What is the probability of getting 9 cards of the same suite in one hand of the game of bridge ?

(*b*) If the moments of a variate X are defined by $E(X^r = 0.6; r = 1, 2, 3 .....)$, show that :

$$P(X = 0) = 0.4, \quad P(X = 1) = 0.6, \quad P(X \geq 2) = 0$$

P.T.O.

(*c*) Six coins are tossed 6400 times. Using Poisson distribution, find the approximate probability of getting six heads *r* times.

(*d*) If X is normally distributed and the mean of X is 12 and the standard deviation is 4, find the probability of $X \leq 20$.

2. Answer the following : 20

(*a*) For any three events A, B and C, prove that :

(*i*) $P(A \cup B | C) = P(A | C) + P(B | C) - P(A \cap B | C)$

(*ii*) $P(A \cap \bar{B} | C) + P(A \cap B | C) = P(A | C)$.

(*b*) The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the Bonus Scheme will be introduced if X, Y and Z becomes managers are 3/10, 1/2 and 4/5 respectively :

(*i*) What is the probability that the Bonus scheme will be introduced ?

(*ii*) If the Bonus Scheme has been introduced, what is the probability that the manager appointed was X ?

3. Answer the following : 20

(*a*) Define cumulants generating function of a random variable. Obtain an expression for first four cumulants in terms of moments.

(*b*) (*i*) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

(*ii*) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability *p* of success in each trial ?

4. Answer the following : 20

(*a*) Obtain the recurrence relation for the central moments of the Poisson distribution. Also obtain the expressions for the moments generating function and the cumulants generating function of the Poisson distribution.

(*b*) The probability of a man hitting a target is 1/4 :

(*i*) If he fires 7 tmes, what is the probability of his hitting the target at least twice ?

(*ii*) How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3 ?

5. Answer the following : 20

(*a*) Define normal distribution. Obtain the expression for the moments generating function and cumulants generating function of normal distribution.

(*b*) The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean 65 and standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70 ?

6. Answer the following : 20

(*a*) A factory produces certain types of outputs by three types of machines. The respectiive daily production figures are :

Machine I : 3000 units; Machine II : 2500 units; Machine III : 4500 units

Past experience shows that 1% of the output produced by Machine I is defective. The corresponding fractions of defectives for the other two machines are 1.2% and 2% respectively. An item is drawn at random from the day's production run and it is found to be defective. What is the probability that it comes from the output of (*i*) Machine I (*ii*) Machine II (*iii*) Machine III ?

(*b*) A manager accepts the work submitted by his typist only when there is no mistake in the work. The typist has to type on an average 20 letters per day about 200 words each. Using Poisson distribution, find the chance of her making a mistake :

(*i*) If less than 1% of the letters submitted by her are rejected.

(*ii*) If on 90% of days all the letters submitted by her are accepted. (Given $e = 2.72$)

This question paper contains 4 printed pages]

RT—385—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper-XI(A)

(Combinatorics)

(Thursday, 2-05-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) How many ways are there to form a three-letter sequence using the letters *a, b, c, d, e, f* :

(*i*) with repetition of letters allowed ?

(*ii*) without repetition of any letter ?

(*iii*) without repetition and containing the letter *e* ?

(*iv*) with repetition and containing *e* ?

P.T.O.

Or

(*b*) Nine students, three from Ms. A's class, three from Mr. B's class, and three from Ms. C's class have bought a block of nine seats for their school's homecoming game. If three seats are randomly selected for each class from the nine seats in a row, what is the probability that the three A students, three B students and three C students will each get a block of three consecutive seats ?

2. Attempt the following : 15

(*a*) Find a generating function for the number of ways to select r doughnuts from five chocolate, five strawberry, three lemon and three cherry doughnuts. Repeat with the additional constraint that there must be at least one of each type.

Or

(*b*) Build a generating function for a_r, the number of selections from :

(*i*) Five red, four black and four white balls.

(*ii*) Five jelly beans, four licorice, eight lollipops with at least one of each type of candy.

3. Attempt the following : 15

(*a*) A bank pays 4 percent interest each year on money on savings accounts. Find recurrence relations for the amount of money a gnome would have after n years if it follows that investment strategies of :

(*i*) Investing Rs. 1,000 and leaving it in the bank for n years.

(*ii*) Investing Rs. 100 at the end of each year.

Or

(*b*) Find a recurrence relation for the number of ways to arrange cars in a row with n spaces if we can use Cadillacs or Hummers or Fords. A Hummer requires two spaces, whereas a Cadillac or a Ford requires just one space.

4. Attempt the following : 15

(*a*) Let $A_1, A_2, ..., A_n$ be n sets in the universal set U. Then with usual notations, prove that

$$N(A_1 \cup A_2 \cup .... \cup A_n) = S_1 - S_2 + S_3 - ..... + (-1)^{k-1} S_k + ... + (-1)^{n-1} S_n$$

P.T.O.

Or

(*b*) Find the number of 4-digit ternary sequences with exactly two 1*s*. Also, find the number with at least two 1*s*.

5. Attempt any *three* of the following : 15

(*a*) How many arrangements are there of the six letters *b, a, n, a, n, a* ?

(*b*) Build a recurrence relation for a_r the number of integer solutions to the equation :

$$e_1 + e_2 + e_3 + e_4 = r, \quad 0 < e_i, e_2, \quad e_4 \text{ odd}, \quad e_4 \leq 3$$

(*c*) Find a recurrence relation for the number of ways to distribte n distinct objects onto four boxes. What is the initial condition ?

(*d*) How many different integer solutions are there to the equation :

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20, \; 0 \leq x_i \leq 8 \; ?$$

This question paper contains **3** printed pages]

RT—334—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper-X

(Complex Analysis-II)

(Monday, 29-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following :

(*a*) State and prove Taylor's theorem. 15

Or

(*b*) (*i*) State and prove Cauchy's inequality.

(*ii*) State and prove Liouville's theorem. 15

2. Attempt the following :

(*a*) Define Removable singularity and state and prove Riemann's theorem. 15

Or

(*b*) Expand the function $f(z) = \dfrac{1}{z^2 + 1}$ in Laurent's series valid in the deleted neighborhood of $z = i, -i$. 15

P.T.O.

3. Attempt the following :

(*a*) State and prove Residue theorem. Evaluate $\int_C f(z)dz$ along different simple closed contours C. 15

Or

(*b*) Show that : 15

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \pi.$$

4. Attempt the following :

(*a*) (*i*) If $f(z)$ is analytic and one-to-one in a domain D, then prove that $f'(z) \neq 0$ in D, so that f is conformal on D.

(*ii*) If $u(z)$ is harmonic in a domain containing the disk $|z - z_0| \leq R$, then prove that $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + Re^{i\theta})d\theta$. 15

Or

(*b*) If $a_n > 0$, $a_n \neq 1$, then prove that the infinite product $\prod_{n=1}^{\infty}(1 - a_n)$ converges if and only if the series $\sum_{n=1}^{\infty} a_n$ converges. Show that $\prod_{n=1}^{\infty}\left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$. 15

5. Attempt any *three* of the following : 5 each

(*a*) Expand the function $f(z) = \log(1 + z)$ in the Maclurin's series.

(*b*) Find the principal part of the Laurent's expansion for the function $f(z) = \frac{z}{z^2+4}$ valid in the neighbourhood of $z = 2i$.

(*c*) If

$$f(z) = \frac{(z+1)\,(z+7)^5 (z-i)^2}{(z^2-2z+2)^4 (z+i)^8 (z-5i)^8},$$

then evaluate $\int_{|z|=2} \frac{f'(z)}{f(z)} dz.$

(*d*) Show that :

$$\prod_{n=1}^{\infty} \left(\frac{n^2-1}{n^3-4} \right) = 4.$$

This question paper contains 4 printed pages]

RT—386—2024

FACULTY OF SCIENCE

M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper XI(B)

(Dynamics and Continuum Mechanics—II)

(Thursday, 02-05-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Define symmetric and antisymmetric tensors and decompose the tensor

$$[T] = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

into a symmetric and antisymmetric tensor. 15

Or

(*b*) Define transpose of a tensor and with usual notations show that : 8

$$\left[\bar{Q}\right]\left[\bar{Q}^T\right] = \left[\bar{Q}^T\right]\left[\bar{Q}\right] = [I].$$

P.T.O.

(*c*) Determine eigenvalue and eigenvectors of the following matrix : 7

$$[T] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

2. (*a*) Let $T = T(t)$ be a tensor valued function of a scalar t, then prove that : 15

(*i*) $\frac{d}{dt}(T + S) = \frac{dT}{dt} + \frac{dS}{dt}$

(*ii*) $\frac{d}{dt}(\alpha(t)T) = \frac{d\alpha}{dt}T + \alpha\frac{dT}{dt}$

(*iii*) $\frac{d}{dt}(TS) = \frac{dT}{dt}S + T\frac{dS}{dt}.$

Or

(*b*) Show that for an orthogonal tensor $Q(t)$, $\left(\frac{dQ}{dt}\right)Q^T$ is an antisymmetric tensor. 8

(*c*) If $\phi = x_1x_2 + 2x_3$, find a unit vector $\bar{n}$ normal to the surface of a constant ϕ passing through the point (2, 1, 0). 7

3. (*a*) Define principal strain and derive expression for principal scalar invariants of strain tensor. 15

Or

(*b*) Given the motion of continuum to be 8

$x_1 = X_1 + ktX_2,\ x_2 = (1 + kt)\ X_2,\ x_3 = X_3$

If the temperature filed is given by the spatial description.

$\Theta = \alpha\ (x_1 + x_2)$, then find $\dfrac{D\Theta}{Dt}$.

(*c*) For the velocity field of 7

$$v_i = \frac{kx_i}{1 + kt}.$$

Find the density of material particle as a function of time.

4\. (*a*) Prove that stress tensor is symmetric. 15

Or

(*b*) Show that for an incompressible fluid, 8

$$\frac{\partial T_{ij}}{\partial X_j} = \frac{\partial p}{\partial X_i} + \mu \frac{\partial^2 v_i}{\partial X_j \partial X_j}.$$

(*c*) State and prove equations of hydrostatics. 7

5\. Attempt any *three* of the following : 15

(*a*) Find the components of stress at a point if the strain matrix is given by

$$[\bar{E}] = 10^{-6} \begin{bmatrix} 30 & 50 & 20 \\ 50 & 40 & 0 \\ 20 & 0 & 30 \end{bmatrix}.$$

P.T.O.

(*b*) If $\phi = x_1^2 + 3x_1x_2 + 2x_3$ is scalar field, then find unit vector normal to the surface at the origin.

(*c*) Verify $[T][T^T] = I = [T^T][T]$ for $T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(*d*) Find the eigenvalues for the rotation tensor R corresponding to a 90° rotation about e_3.

This question paper contains 4 printed pages]

RT—54—2024

FACULTY OF SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–VII

(Linear Algebra)

(Thursday, 18-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(*a*) If v be a vector space with dimension n, then prove that :

(*i*) Any finite generating set for v contains at least n vectors, and a generating set for v that contains exactly n vectors is a basis for v.

(*ii*) Any linearly independent subset of v that contains exactly n vectors is a basis for v.

(*b*) If v be the vector space generated by a finite set S. Then prove that some subset of S is a basis for u. Hence v has a finite basis. Furthermore prove that the set $\{1, x, x^2, ... x^n\}$ is a basis for $P_n(F)$.

P.T.O.

2. Attempt any *one* of the following : 15

(*a*) If V and W be vector space of equal (finite) dimension, and $T : V \rightarrow W$ be linear, then the following are equivalent :

(*i*) T is one-to-one

(*ii*) T is onto

(*iii*) rank (T) = dim(V)

(*b*) State and prove dimension theorem.

3. Attempt any *one* of the following : 15

(*a*) If A be an $m \times n$ matrix and P and Q are invertible $m \times m$ and $n \times n$ matrices, respectively, then :

(*i*) rank (AQ) = rank(A),

(*ii*) rank (PA) = rank (A) and therefore,

(*iii*) rank (PAQ) = rank (A).

(*b*) If T be a linear operator on a vector space v, and $\lambda_1, \lambda_2, \ldots\ldots, \lambda_k$ be distinct eigenvalues of T and $v_1, v_2, \ldots\ldots, v_k$ are eigenvectors of T such that λ_i corresponds to v_i $(1 \leq i \leq k)$, then $\{v_1, v_2, \ldots\ldots, v_k\}$ is linearly independent.

4. Attempt any *one* of the following : 15

(*a*) If v be an inner product space over F with for all x, $y \in v$ and C $\in$ F, then the following statements are true :

(*i*) $\| cx \| = | c | . \| x \|$

(*ii*) $| \langle x, y \rangle | \le \| x \| . \| y \|$.

(*iii*) $\| x + y \| \le \| x \| + \| y \|$

(*b*) If v be an inner product space and T and U be linear operators on v, then :

(*i*) $(T + U)^* = T^* + U^*$

(*ii*) $(CT)^* = \bar{C}T^*$ for any $C \in F$

(*iii*) $(TU)^* = U^* T^*$.

5. Attempt any *three* of the following : 15

(*a*) If v be an inner product space, then prove that :

$$\| x + y \|^2 + \| x - y \|^2 = 2 \| x \|^2 + 2 \| y \|^2 \; \forall x, y \in v$$

(*b*) If T is linear operator on a vector space v, then prove that N(T) is invariant under T.

P.T.O.

(*c*) If $T : R^2 \to R^3$ defined by

$T(a_1, a_2) = (2a_1 - a_2, 3a_1 + 4a_2, a_1)$

then prove that T is linear transformation.

(*d*) If S = {(1, 2, – 1), (1, 0, 2), (2, 1, 1)}, then determine whether S is basis for R^3 or not ?

This question paper contains **3** printed pages]

RT—133—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–VIII

(Measure and Integration Theory)

(Saturday, 20-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) Prove that every interval is a measurable set. Show that every countable set has measure zero. Does the converse hold ? Justify your answer.

Or

(*b*) State and prove Lebesgue's dominated convergence theorem. Also, show that the constant function is measurable.

P.T.O.

2. Attempt the following : 15

(*a*) Let $f \in BV[a, b]$, then prove that : $f(b) - f(a) = P - N$ and $T = P + N$, where all variations being in the finite interval $[a, b]$. Also, let f be defined by $f(x) = |x|$. Find the four derivative at $x = 0$.

Or

(*b*) If $f \in L(a, b)$, then prove that $\int_a^x f(t)\,dt$ is a constinuous function on $[a, b]$ and $f \in BV\ [a, b]$.

3. Attempt the following : 15

(*a*) Show that, $H(R) = \left\{E ! E \subseteq \bigcup_{n=1}^{\infty} E_n, E_n \in R\right\}$.

Or

(*b*) Let μ^* be an outer measure on H(R) and let S^* denote the class of μ^* measurable sets. Then prove that S^* is a σ-ring and μ^* restricted to S^* is a complete measures.

4. Attempt the following : 15

(*a*) Let υ be a signed measure on $[[X,S]]$, the prove that there exists a positive set A and a negative set B such that : $A \cup B = X$, $A \cap B = \phi$.

The pair A, B is said to be a Hahn decomposition of X with respect to v. Also prove that it is unique upto the extent that if A_1, B_1 and A_2, B_2 are Hahn decomposition of X with respect to v, then $A_1 \Delta A_2$ is an v-null set.

Or

(*b*) Show that, a countable union of sets positive with respect to a signed meausre v is positive set. Also, show that if :

$$\phi(E) = \int_E f d\mu,$$

Where, $\int f d\mu$ is defined, then ϕ is a signed measure.

5. Attempt any *three* of the following :

(*a*) Show that if f and g are measurable, $|f| \leq |g|$ a.e., and g is integrable, then f is integrable.

(*b*) Give an example to show that, $D^+ (f + g) \neq D^+ f + D^+ g$.

(*c*) Let $f = g$ *a.e* (μ), where μ is a complete measure. Show that if if is measurable, so is g.

(*d*) Show that, if v_1, v_2 and μ are measures and $v_1 \perp \mu$, $v_2 \perp \mu$, then $v_1 + v_2 \perp \mu$.

This question paper contains **3** printed pages]

RT—222—2024

FACULTY OF SCIENCE

M.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper IX

(Partial Differential Equations)

(Tuesday, 23-04-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Prove that A necessary and sufficient condition that the Pfaffian differential equation : 15

$$\bar{X} \cdot \overline{dr} = P(x, y, z)\, dx + Q(x, y, z)\, dx + R(x, y, z)\, dz = 0 \text{ be}$$

integrable that $(\bar{X} \cdot \text{Curl } \bar{X}) = 0$.

Or

(*b*) Find the general solution of : 15

$$x(y^2 - z^2)\, p - y(z^2 + x^2)\, q = z(x^2 + y^2).$$

P.T.O.

2. (*a*) Prove that the necessary and sufficient condition for the integrability of equation : 15

$dz = \phi(x, y, z)\, dx + \psi(x, y, z) dy$ is :

$$[f, g] = \frac{\partial(f, g)}{\partial(x, p)} + p\frac{\partial(f, g)}{\partial(z, p)} + \frac{\partial(f, g)}{\partial(y, q)} + q\frac{\partial(f, g)}{\partial(z, q)} = 0.$$

Or

(*b*) Find the complete integral by Charpit's method : 15

(*i*) $f = x^2p^2 + y^2q^2 - 4 = 0$

(*ii*) $(p^2 + q^2)y - qz = 0.$

3. (*a*) Reduce the partial differential equation $u_{xx} - x^2u_{yy} = 0$ to a canonical form. 15

Or

(*b*) Derive an expression for vibrations of semi-infinite string. 15

4. (*a*) State and prove Harnack's theorem. 15

Or

(*b*) Reduce the equation to the canonical form and solve it : 15

$$y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y.$$

5. Attempt any *three* of the following :

(*a*) Form the partial differential equation : 5

$$(x - a)^2 + (y - b)^2 + z^2 = 1.$$

(*b*) Find the general solution of $xp + yq = z$. 5

(*c*) Find the complete integral of $p^2 + q^2 = x + y$. 5

(*d*) Show that $u = f(x + t)$ is a solution of the PDE $u_{xx} - u_{tt} = 0$. 5

This question paper contains **3** printed pages]

RT—181—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper XV-(A)

(Analytical Number Theory)

(Monday, 22-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

Note :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) State and prove Fermat's theorem. Also, solve $18x \equiv 30 \pmod{42}$.

Or

(*b*) If n is odd pseudoprime, then prove that $M_n = 2^n - 1$ is a large one. Also, give an example to show $a^2 \equiv b^2 \pmod{n}$ does not implies $a \equiv b \pmod{n}$.

P.T.O.

2. Attempt the following : 15

(*a*) If p is an prime number and $d|p - 1$, then the congruence :

$$x^d - 1 \equiv 0 \pmod{p}$$

has exactly d solutions. Also, find the order of integers 2, 3, 5 :

(*i*) Modulo 17

(*ii*) Modulo 19.

Or

(*b*) For $k \geq 3$, prove that 2^k has no primitive roots. Also, find the four primitive roots of 26.

3. Attempt the following : 15

(*a*) State and prove quadratic reciprocity law. Also, show that 3 is a quadratic residue of 23, but a non-residue of 31.

Or

(*b*) Let p be an odd prime and $gcd(a, p) = 1$, then prove that a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1 \pmod{p}$.

Also, use Gauss-Lemma to compute each of Legendre symbols :

(*i*) (8/11)

(*ii*) (5/19)

4. Attempt the following : 15

(*a*) State and prove Selberg identity. Also, define Mangoldt function $\Lambda(n)$ and find the values of Mangoldt function for $n = 1$ to 10.

Or

(*b*) Let f be multiplicative, then prove that, if f is completely multiplicative if and only if :

$$f^{-1}(n) = \mu(n) \cdot f(n) \text{ for all } n \geq 1.$$

Also, find :

(*i*) $\phi(343)$

(*ii*) $\phi(4300)$.

5. Attempt any *three* of the following : 15

(*a*) Show that 41 divides $2^{20} - 1$.

(*b*) Verify that 3 is a primitive root of 7.

(*c*) Find the values of the following legendre symbols (19/23) and (−23/59).

(*d*) Find all integer n such that :

$$\Phi(n) = n/2.$$

This question paper contains **2** printed pages]

RT—277—2024

FACULTY OF SCIENCE

M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper XVI

(Fluid Mechanics–I)

(Wednesday, 24-4-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) Each question carries equal marks.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(*a*) Define real fluid. Derive an equation of continuity for steady incompressible fluid.

Or

(*b*) Show that the fluid of constant density can have the velocity $\bar{q} = \left[\frac{-2xyz}{(x^2+y^2)^2}, \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{y}{x^2+y^2}\right]$. Find the vorticity vector.

2. Attempt any *one* of the following : 15

(*a*) Explain the mechanism of Pittot tube.

Or

(*b*) Prove that, in fluid region the pressure is same in all direction.

P.T.O.

3. Attempt any *one* of the following : 15

(*a*) State and prove Kelvin's theorem.

Or

(*b*) Prove that the Kinetic energy of sphere moving with constant velocity in liquid which is at rest is (1/4)M'U^2.

4. Attempt any *one* of the following : 15

(*a*) Describe the irrotational motion of an incompressible liquid for which the complex potential is w = ikLogZ.

Or

(*b*) Derive an expression for stream function.

5. Attempt any *three* of the following : 15

(*i*) Derive an expression for acceleration of fluid.

(*ii*) Define streamtube, pressure and doublet.

(*iii*) Derive Bernoulli's equation.

(*iv*) Write a note on doublet.

This question paper contains **4** printed pages]

RT—18—2024

FACULTY OF ARTS/SCIENCES

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–XIII

(Functional Analysis)

(Monday, 16-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following :

(*a*) (*i*) State and prove closed graph theorem. 8

(*ii*) State and prove Open Mapping Theorem. 7

Or

(*b*) Suppose N is a normed linear space and M is closed linear subspace of N. Prove that $\{x + \mathrm{M} \mid x \in \mathrm{N}\}$ is normed linear space. Moreover prove that $\frac{\mathrm{N}}{\mathrm{M}}$ is a Banach space with respect to the norm defined as $\|x + \mathrm{M}\|' = \inf\left\{\|x + m\| \mid m \in \mathrm{M}\right\}$ if N is Banach space. 15

P.T.O.

2. Attempt the following :

(*a*) (*i*) State and prove Riesz Representation Theorem 8

(*ii*) State and prove Bessel's inequality. 7

Or

(*b*) If H is Hilbert space and $\{x, y\}$ is the inner product defined on H, then $\forall\ x, y \in$ H prove that :

(*i*) $\|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2 = 4\left[\langle x, y\rangle\right]$

(*ii*) $\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2$ 15

3. Attempt the following :

(*a*) (*i*) Prove that a closed linear subspace M of H is invariant under T if and only if $M^{\perp}$ is invariant under T^*. 8

(*ii*) Prove that P is projection on closed linear subspace M of H, then M is invariant T if any and if TP = PTP. 7

Or

(*b*) If $T^* \in B(H)$, then prove that the following :

(*i*) $(T_1 + T_2)^* = T_1^* + T_2^*$.

(*ii*) $(\alpha T)^* = \bar{\alpha} T^*$.

(*iii*) $(T_1.T_2)^* = T_2^*.T_1^*$.

(*iv*) $(T^*)^* = T^{**} = T$.

(*v*) $\| T \| = \| T^* \|$. 15

4. Attempt the following :

(*a*) Suppose T is an arbitrary operator on finite dimensional Hilbert space H. $\lambda_1, \lambda_2, ... \lambda_n$ are corresponding eigen values of T corresponding to the eigen vectors $x_1, x_2, ... x_n$. Suppose $M_1, M_2, ... M_n$ are the corresponding eigen spaces and $P_1, P_2 .... P_n$ are the projections on these eigen spaces :

(*i*) If M_i's are pairwise orthogonal and spans H, then prove that P_i's are pairwise orthogonal and $I = \sum_{i=1}^{n} P_i$ and $T = \sum_{i=1}^{n} \lambda_1 P_i$.

(*ii*) If T is normal, then prove that each M_i spans H. 15

Or

(*b*) Suppose T is an arbitrary operator on finite dimensional Hilbert space H. $\lambda_1, \lambda_2 ... \lambda_n$ are corresponding eigen values of T corresponding to the eigen vectors $x_1, x_2, ... x_n$. Suppose $M_1, M_2, ... M_n$ are the corresponding eigen spaces and $P_1, P_2, ... P_n$ are the projections on these eigen spaces : 15

(*i*) If P_i's are pairwise orthogonal and $I = \sum_{i=1}^{n} P_i$ and $T = \sum_{i=1}^{n} \lambda_i P_i$, then prove that T is normal.

(*ii*) If T is normal, then prove that M_i's are pairwise orthogonal.

15

P.T.O.

5. Attempt any *three* of the following : 5 marks each

(*a*) Show that vector addition is jointly continuous.

(*b*) If M is linear subspace of H, then show that M is closed if any only if $M = M^{\perp\perp}$.

(*c*) If $T \in B(H)$ is non-singular operator, then prove that T^* is also non-singular operator moreover prove that $(T^*)^{-1} = (T^{-1})^*$.

(*d*) Define, (*i*) Eigen Value, (*ii*) Eigen Vector, (*iii*) Eigen Space.

This question paper contains **5** printed pages]

RT—183—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper XV-C

(Fuzzy Sets and their Applications-I)

(Monday, 22-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

Note :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define fuzzy sets with suitable examples and explain why we need fuzzy set theory. Whether the law of contradiction and the law of exclusive middle are valid in fuzzy set theory. Justify your claim. 15

Or

The crisp universal set X of ages that we have selected is :

$$X = \{5, 10, 20, 30, 40, 50, 60, 70, 80\}$$

P.T.O.

and the fuzzy set labelled as infant, adult, young and old with the following membership grade :

Elements	**Infant**	**Adult**	**Young**	**Old**
05	0	0	1	0
10	0	0	1	0
20	0	0.8	0.8	0.1
30	0	1	0.5	0.2
40	0	1	0.2	0.4
50	0	1	0.1	0.6
60	0	1	0	0.8
70	0	1	0	1
80	0	1	0	1

expressed above table in graphical form. Find supp (young) and α-cut for α = 0.2, 0.8, 1 for young.

2. Prove that following : 15

(*i*) If c is a continuous fuzzy complement, then c has a unique equilibrium.

(*ii*) If a complement c has an equilibrium e_c, then prove that $d_{e_c} = e_c$

Or

Write the set of axioms of the axiomatic skeleton for fuzzy set unions and prove that :

$$\lim_{w \to \infty} \min [1, (a^w + b^w)^{1/w}] = \max(a, b) .$$

3. Explain the concept of fuzzy relation, kinds of fuzzy relation and fuzzy matrix with their suitable examples. 15

Or

Define transitive closure of a crisp relation R(X, X) and write algorithm for determine it, using algorithm determine the transitive max-min closure $R_T(X, X)$ for fuzzy relation R(X, X) defined by :

$$M_R = \begin{bmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

4. Draw the membership matrix and partition tree for the following fuzzy relation R(X, X) is a similarity relation on : 15

$$X = \{a, b, c, d, e, f, g\}.$$

The level set of R is :

$$\Lambda_R = \{0, 0.4, 0.5, 0.8, 0.9, 1\}$$

and R is associated with a sequence of five nested partitions $\pi(R_\alpha)$ for $\alpha \in \Lambda_R$ and $\alpha > 0$.

Or

Define morphisms in fuzzy relation. Explain ordinary fuzzy homomorphism and strong homomorphism with suitable examples.

5. Attempt any *three* out of *four* : 15

(*a*) Consider the matrix equation :

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} 0 \begin{bmatrix} .9 & .5 \\ .7 & .8 \\ 1 & .4 \end{bmatrix} = \begin{bmatrix} .6 & .3 \\ .2 & 1 \end{bmatrix}$$

whose general form is :

$$[p_{i,\,j}]\ 0\ [q_{j,k}] = [r_i,\, k)$$

where

$$i \in \mathbb{N}_2,\ j \in \mathbb{N}_3 \text{ and } k \in \mathbb{N}_2,$$

then show that the given matrix has no solution.

(*b*) Represent the following fuzzy relation R(X, X) in simple and sagittal diagram :

X	Y	μ_R(X, Y)
1	1	0.7
1	3	0.3
2	2	0.7
2	3	1
3	1	0.9
3	4	1
4	3	0.8
4	4	0.5

(*c*) Prove that every fuzzy complement has atmost one equilibrium.

(*d*) Compute the scalar cardinality for each of the following fuzzy sets :

(*i*) $A = 0.4/v + 0.2/w + 0.5/x + 0.4/y + 1/z$

(*ii*) $B = 1/x + 1/y + 1/z$.

This question paper contains 4 printed pages]

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FACULTY OF SCIENCE

M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper XVII(A)

(Integral Transform-I)

(Tuesday, 30-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) If f is piecewise continuous on $t \geq 0$ and is $\mathrm{O}\left(e^{c_0 t}\right)$, then prove that : 15

(*i*) $\mathrm{L}\left[\int_0^t f(u)du\right] = \frac{\mathrm{F}(p)}{\mathrm{P}}$

(*ii*) $\mathrm{L}\left[t^n f(t)\right] = (-1)^n \mathrm{F}(p), n = 1, 2, ...$

Or

(*b*) State and prove convolution theorem for Laplace transform. 8

(*c*) Find :

$$\mathrm{L}^{-1}\left[\frac{2}{(p+1)\left(p^2+1\right)}; t\right].$$ 7

P.T.O.

2. (*a*) Using Laplace transform solve :

$$u_{xx} = a^{-2}u_t,\ 0 < x < \infty,\ t > 0$$

B.C. $u(0, t) = f(t),\ u(x, t) \to 0$, as $x \to \infty$

I.C. $u(x, 0) = 0,\ 0 < x < \infty.$ 15

Or

(*b*) Solve the I.V.P.

$$y'' - 6y' + 9y = t^2 e^{3t},\ \ y(0) = 2,\ y'(0) = 6.$$ 8

(*c*) If F(p) is Laplace transform of $f(t)$, then with usual notations prove that :

$$\mathrm{L}\left[f(t-a)\, h(t-a);\ p\right] = e^{-ap}\ \mathrm{F}(p),\ \ a > 0.$$ 7

3. (*a*) If $f(t)$ and F(s) are Fourier transform pairs, then show that : 15

(*i*) $\mathbf{F}\left\{e^{iat} f(t); \mathrm{S}\right\} = \mathrm{F}(\mathrm{S} + a)$

(*ii*) $\mathbf{F}\left\{f(t-a); \mathrm{S}\right\} = e^{ias}\ \mathrm{F}(\mathrm{S})$

(*iii*) $\mathbf{F}\left\{f'(t); \mathrm{S}\right\} = -is\ \mathrm{F}(\mathrm{S}).$

Or

(*b*) Evaluate the integral :

$$\int_0^\infty \frac{dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)}.$$ 8

(*c*) If f is absolutely integrable and piecewise smooth, if $t^m f(t)$ has a Fourier transform then prove that :

$$\mathbf{F}\left\{t^m f(t); s\right\} = (-i)^m \, \mathrm{F}^{(m)}(s), \; m = 1, 2......$$ 7

4. (*a*) Using Fourier transform solve :

$$u_{xx} = a^{-2} u_t,$$

B.C. $u(0, t) = 0, \; u(x, t) \to 0, \text{ as } x \to \infty$

I.C. $u(x, 0) = f(x), \; 0 < x < \infty.$ 15

Or

(*b*) Use Fourier transform to solve : 8

$$y'' - y = -h\left(1 - |x|\right), \; -\infty < x < \infty$$

$$y(x) \to 0, \; y'(x) \to 0 \text{ as } |x| \to \infty.$$

(*c*) Solve :

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |x - \xi|^{-1/2} \, u(\xi) \, d\xi.$$ 7

5. Attempt any *three* of the following : 15

(*a*) Find Mellin transform of

$$\frac{1}{(1 + ax)^m}, \; m > 0.$$

P.T.O.

(*b*) Evaluate :

$$\int_0^\infty \frac{e^{-3t} - e^{-6t}}{t}\, dt.$$

(*c*) Find $L\{\sin at;\ p\}$ and $L\{\cos at;\ p\}$.

(*d*) If $a \neq 0$, then

$$\mathbf{F}\{f(at);\ s\} = \frac{1}{|a|}\, F\left(\frac{s}{a}\right).$$

This question paper contains **3** printed pages]

RT—98—2024

FACULTY OF SCIENCE

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper XIV

(Topology)

(Friday, 19-4-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(*a*) If β be the basis for the topology on X, then prove that :

$$\beta_Y = \{B n Y \mid B \in \beta\}$$

is a basis for the subspace topology on Y.

Also if Y is open subspace of X and U is open in Y, then prove that U is open in X.

P.T.O.

(*b*) If β and β^1 are basis for the topologyes τ and τ' respectively on X. Then prove that the following are equivalent :

(*i*) τ' is finer, than τ.

(*ii*) For each $x \in X$ and each

basis element $\beta \in \beta$ containing *x*, there exist a basis element $B' \in \beta'$ such that $x \in \beta' \subseteq B$.

2. Attempt any *one* of the following : 15

(*a*) Let X and Y are any topological spaces. If $f : X \to Y$ be a function, then prove that the following are equivalent :

(*i*) *f* is continuous

(*ii*) for every subset A of X, one has $f(\bar{A}) \subseteq \overline{f(A)}$.

(*iii*) if B is closed in Y, then $f^{+}(B)$ is closed in X.

(*b*) Prove that image of a connected space under continuous function is connected. Also, if X is a locally connected space then prove that, as U is open in X, then each component of U is open in X.

3. Attempt any *one* of the following : 15

(*a*) P.T. product of finitely many compact spaces is compact.

(*b*) State and prove uniform continuity theorem.

4. Attempt any *one* of the following : 15

(*a*) Define normal space and prove that every metrizable space is a normal space.

(*b*) If one point sets in X are closed in X, then prove that X is a regular topological space iff for a given point x of X and neighbourhood U of x there is a neighbourhood V of x such that :

$$\bar{V} \subseteq U.$$

5. Attempt any *three* of the following : 15

(*a*) Prove that subspace of second countable space is a second countable space.

(*b*) Prove that every closed subspace of a compact space is compact.

(*c*) Prove that composition of two continuous functions is continuous.

(*d*) If Y be a subspace of X and $\bar{A}$ denote the closure of A in X, then prove that $\bar{A} \cap Y$ be the closure of A in Y".

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FACULTY OF SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–XX

Abstract Algebra–II (Field Theory)

(Saturday, 20-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(*a*) State and prove Eisenstein's criterion.

(*b*) If $f(x) \in Z[x]$ be primitive polynomial, then prove that $f(x)$ is reducible over Q iff $f(x)$ is reducible over Z.

2. Attempt any *one* of the following : 15

(*a*) Prove that any finite field F with p^n elements is the splitting field of $x^{p^n} - x \in F_p[x]$. Consequently any two finite fields with p^n elements are isomorphic.

P.T.O.

(*b*) If $f(x)$ be an irreducible polynomial over F, then $f(x)$ has a multiple root iff $f'(x) = 0$.

3. Attempt any *one* of the following : 15

(*a*) State and prove Dedekind lemma.

(*b*) If E be a finite separable extension of a field F, then prove that the following are equivalent :

(*i*) E is normal extension of F

(*ii*) F is the fixed field of G (E/F).

4. Attempt any *one* of the following : 15

(*a*) If a and b are constructable number, then prove that $a + b$, $a - b$ and $\sqrt{a}$ $(a > 0)$ are constructable.

(*b*) If F be a field and U be a finite subgroup of multiplicative group $F^* = F - \{0\}$, then U is cyclic.

5. Attempt any *three* of the folowing : 15

(*a*) Show that it is impossible to construct a regular 9–90n or 7–90n using rular and compass.

(*b*) Prove that Galois group of the polynomial $f(x) = x^5 - 1, x \neq 1$ is isomorphic to cyclic group of order 4.

(*c*) Prove $Q\left(\sqrt{3}, \sqrt{5}\right) = Q\left(\sqrt{3} + \sqrt{5}\right)$.

(*d*) If E be an extension of a field F, and $\alpha \in E$ be algebraic over F, then α is separable over F iff $F(\alpha)$ is a separable extension of F.

This question paper contains **3** printed pages]

RT—225—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper-XXI(A)

(Classical Mechanics)

(Tuesday, 23-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) Derive the Lagrange's equation of motion from D'Alembert's principle. Also, find the equation of kinetic energy of rotating rigid body.

Or

(*b*) Define the following :

(*i*) Scleronomic constraint

(*ii*) Rheonomic constraint

(*iii*) Generalised co-ordinates

(*iv*) Degree of freedom.

and give one example of each. Also, find the equation of motion of compound pendulum.

P.T.O.

2. Attempt the following : 15

(*a*) State and prove the principle of least action.

Or

(*b*) Derive Hamilton's canonical equation of motion. Also, if Lagrangian L is scleronomic, then prove that Hamiltonian H represents the total energy.

3. Attempt the following : 15

(*a*) State and prove the fundamental lemma of calculus of variation. Also, find the stationary value of the function :

$$\int_0^{\pi/2} [y'^2 - y^2 + 2xy]\, dx$$

with $y(0) = 0$ and $y(\pi/2) = 0$.

Or

(*b*) Discuss the case functional F does not depend on y. Show that the shortest curve joining two fixed points is straight line.

4. Attempt the following : 15

(*a*) Explain Brachstocrone problem and find its extremal.

Or

(*b*) Define geodesic. Show that the geodesic on a right circular cylinder is Helix.

5. Attempt any *three* of the following : 15

(*i*) Obtain an equation of motion in Linear Harmonic Oscillator.

(*ii*) Prove that the generalised momentum conjugate to a cyclic co-ordinate is conserved.

(*iii*) Find the extremal of functional

$$J[y(x)] = \int_1^2 \frac{\sqrt{1+y^2}}{x} dx$$

with $y(1) = 0$, $y(2) = 1$.

(*iv*) Discuss the invariance of Euler's equation.

This question paper contains **2** printed pages]

RT—338—2024

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–XXII

(Fluid Mechanics-II)

(Monday, 29-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) Each question carries equal marks.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following : 15

(*a*) State and prove Milne-Thomson circle theorem.

Or

(*b*) A source and sink of equal strength are placed at points $(a, 0)$ and $(-a, 0)$ respectively within the fixed circular boundary [Z] = $2a$. Show that streamlines are given by :

$(r^2 - 16a^2)(r^2 - a^3) - 16a^2y^2 = \lambda y (r^2 - 4a^2)$. Where K is constant.

P.T.O.

2. Attempt any *one* of the following : 15

(*a*) Explain the reservoir discharge through a channel of varying section.

Or

(*b*) Prove that the Isentropic gas relation $\frac{T_0}{T} = \left(\frac{\rho_0}{\rho}\right)^{\gamma-1}$

3. Attempt any *one* of the following : 15

(*a*) Derive the relation between stress and rate of strain.

Or

(*b*) Derive an expression for translational motion of fluid element.

4. Attempt any *one* of the following : 15

(*a*) Show that the velocity profile for flow between two parallel planes is parabolic.

Or

(*b*) Describe the Diffusion of vorticity.

5. Attempt any *three* of the following : 15

(*a*) Write the solution of Wave equation in spherical coordinate system

(*b*) Illustrate the stress components on a real fluid

(*c*) Write a note on Prandtl's boundary layer

(*d*) Find the profile φ (x, t) of a one-dimensional wave propagation if at $t = 0$, $\phi = F(x)$, $\frac{\partial\theta}{\partial t} = G(x)$.

This question paper contains **3** printed pages]

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FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper XXI-C

(Fuzzy Sets and Their Ap plications-II)

(Tuesday, 23-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Given a consonant body of evidence (**F**, m) then prove that the associated constant belief and plausibility measures possess the following property : 15

(*i*) Bel $(A \cap B) = \min[\text{Bel}(A), \text{Bel}(B)]$ for all A, B $\in$ **P**(X);

(*ii*) $\text{Pl}(A \cup B) = \max[\text{Pl}(A), \text{Pl}(B)]$ for all A, B $\in$ **P**(X).

Or

Define a fazzy measure function in detail and explain axioms boundary conditions monotonicity continuity with statement.

2. Define measure of fuzziness, degree of fuziness and the term maximally fuzzy in f(A). Assume the maximum value if and only if A is maximally fuzzy, by the membership grade $\frac{1}{2}$ for all $x \in X$. 15

Or

Prove that A belief measure Bel on a finite power set **P**(X) is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = \text{Bel}\ \{x\})$ and $m(A) = 0$ for all subsets of X that are not singletons.

P.T.O.

3. Let $U : \mathbf{R} \rightarrow [0, \infty)$, where $\mathbf{R}$ denotes the set of all ordere possibility distributions be a function such that $U(r)$ is supposed to characterize the amount of uncertainly associated with the possibility distribution r. Explain the term expansibility, subadditivity, additivity, continuity, monotonicity, minimum, maximum, branching and normalization for U. 15

Or

Consider universal set X, three non-empty subsets of which are of our interest : A, B and $A \cap B$ assume that the only evidence is expressed in terms of the total belief focusing on A and the total belief focusing on B. The aim is to estimate the basic assignment values for X, A, B and $A \cap B$. The use of the principle of maximun non-specificity leads in this case to the following optimization problem.

Determine values $m(X)$, $m(A)$, $m(B)$ and $m(A \cap B)$ for which the function $m(X)\log_2 |x| + m(A)\log_2 |A| + m(B)\log_2 |B| + m(A \cap B) \log_2 |A \cap B|$ reaches its maximum subject to the contraints :

$$m(A) + m(A \cap B) = a$$

$$m(B) + m(A \cap B) = b$$

$$m(X) + m(A) + m(B) + m(A \cap B) = 1$$

$m(X), m(A), m(B), m(A \cap B) \geq 0$, where $a, b \in [0, 1]$ are given numbers.

4. Give the difference between crisp and fuzzy data for two variables each with three states (values, labels) 0, 1, 2. Define pseudo frequencies and derive expression for pseudo frequencies. 15

Or

Justify the fuzzy model of decision making proposed by Bellman and Zadeh with a suitable illustrated example.

5. Attempt any *three* out of the four : 15

(*a*) Write a short note on 'fuzzy data base model' and its associated fuzzy relational algebra with suitable example.

(*b*) Write a short note on 'Hartley information'.

(*c*) Prove that :

$$H(X|Y) = H(X, Y) - H(Y).$$

(*d*) Derive the formula for two conditional probability distributions $P_{X|Y}$ and $P_{Y|X}$ and prove that the set X and Y are independent of each other if and only if they are non-interactive.

This question paper contains **4** printed pages]

RT—391—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper-XXIII (A)

(Integral Equations)

(Thursday, 2-05-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following : 15

(*a*) Explain the method of solving the equation :

$$y(x) = \lambda \int_a^b k(x, t)\, y(t)\, dt$$

where $k(x, t)$ is separable. Hence solve :

$$y(x) = \lambda \int_0^1 e^x e^t\, y(t)\, dt$$

P.T.O.

(*b*) Reduce $y''(x) - \lambda\, y(x) = 0$ with initial conditions $y(0) = 0$, $y(l) = 0$ to a Fredholm integral equation. Conversely, recover the original differential equation with the initial conditions from the integral equation obtained.

2. Attempt the following : 15

(*a*) Solve :

$$y(x) = f(x) + \lambda \int_0^1 (x + t)\, y(t)\, dt$$

Or

(*b*) Solve :

$$y(x) = \cos x + \lambda \int_0^\pi \sin(x - t)\, y(t)\, dt$$

3. Attempt the following : 15

(*a*) Prove that the multiplicity of any non-zero eigenvalue is finite for every symmetric kernel for which

$$\int_a^b \int_a^b |k(x, t)|^2\, dx\, dt$$

is finite.

Or

(*b*) Prove that the eigenfunctions of a symmetric kernel of a Fredholm integral equation of the first kind, corresponding to different eigenvalues are orthogonal.

4. Attempt the following : 15

(*a*) Solve the Volterra integral equation of the first and second kind with convolution type kernel using Laplace transform. Prove that if the kernel of the Volterra integral equation is of convolution type, then its resolvent kernel is also of convolution type. Using Laplace transform, obtain the expression for the resolvent kernel of Volterra integral equation of second kind with convolution type kernel.

Or

(*b*) Solve the general Abel singular integral equation :

$$f(x) = \int_0^x \frac{y(t)}{[h(x) - h(t)]^{\alpha}}\, dt, \quad 0 < \alpha < 1$$

where $h(x)$ is a strictly monotonically increasing and differentiable function known function in (a, b) and $h'(t) \neq 0$. Hence solve the equation :

$$f(x) = \int_0^x \frac{y(t)}{(\cos x - \cos t)^{\frac{1}{2}}}\, dt, \quad 0 \le a < x < b \le \pi$$

5. Attempt any *three* of the following : 15

(*a*) Convert the initial value problem into an integral equation :

$y'' + y = 0, \quad y(0) = 0, \quad y'(0) = 0$

P.T.O.

(*b*) Solve the integral equation :

$$g(s) = s + \int_0^1 s u^2 \, g(u) \, du$$

(*c*) Let the sequence $\{\phi_k(x)\}$ be all the eigenfunctions of a symmetric $\pounds_2$-kernel with $\{\lambda_k\}$ as the sequence of the corresponding eigenvalues. Then prove that the series $\Sigma_{n-1}^{\infty} \frac{|\phi_n(x)|^2}{\lambda_n^2}$ converges and its sum is bounded by C_1^2, which is an upper bound of the integral $\int_a^b |k^2(x, t)| \, dt$, where k is the kernel of the Fredholm integral equation :

$$y(x) = f(x) + \lambda \int_a^b k(x, t) \, y(t) \, dt$$

(*d*) Using Laplace transform, find the resolvent kernel of the intergral equation :

$$Y(t) = F(t) + \int_a^b (t - x) \, Y(x) \, dx$$

This question paper contains 4 printed pages]

RT—57—2024

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New/CBCS Pattern)

MATHEMATICS

Paper–XIX

(Numerical Analysis)

(Thursday, 18-04-2024) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Scientific calculator is allowed.

1. Attempt the following :

(*a*) Explain in detail Chebyshev method of solving the equation $f(x) = 0$. Also find the rate of convergence of Chebyshev method. 15

Or

(*b*) Obtain the smallest positive root of the equation $f(x) = \cos x - xe^x = 0$ by using Newton-Raphson method. Perform four iterations. 15

P.T.O.

2. Attempt the following :

(*a*) Define :

(1) Band matrix

(2) Permutation matrix. Give *one* example.

Explain in detail Gauss elimination method of solving the system of equations $Ax = b$. 15

Or

(*b*) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

by using LU Decomposition method $u_{11} = u_{22} = u_{33} = 1$. 15

3. Attempt the following :

(*a*) (*i*) Discuss in detail Gauss-Seidel iteration method of solving the system of equations $Ax = b$. Also obtain its error format.

(*ii*) State and prove Brauer theorem. 15

Or

(*b*) For the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(*i*) Find all the eigenvalues and the corresponding eigenvectors.

(*ii*) Verify that $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigenvector. 15

4. Attempt the following :

(*a*) Let function $f(x)$ be continuous on $[a, b]$ and its values are known at $n + 1$ distinct points $a \leq x_0 < x_1 < x_2 < \ldots\ldots < x_{n-1} < x_n \leq b$ of $[a, b]$, then prove that there exists a unique polynomial $P(x)$ which satisfies the conditions $P(x_i) = f(x_i) \; \forall \; i = 0, 1, 2 \ldots\ldots n$ if Vandermonde's determinant is non-zero. 15

Or

(*b*) (*i*) Construct the divided difference table for the data :

x	0.5	1.5	3.0	5.0	6.5	8.0
$f(x)$	1.625	5.875	31.0	131.0	282.125	531.0

Hence find the interpolating polynomial and also find the approximate value of $f(7)$.

(*ii*) Calculate the nth divided difference of $\frac{1}{x}$ based on the points $x_0, x_1, \ldots\ldots, x_n$. 15

P.T.O.

5. Attempt any *three* of the following : 3×5=15

(*a*) Perform four iterations of the Newton-Raphson method to obtain the approximate value of $(17)^{1/3}$. Take the initial approximation as $x_0 = 2$.

(*b*) Show that the decomposition method fails to solve the system of equations :

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

(*c*) Determine the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

by using iterative method. Given Approximate inverse is :

$$B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$$

(*d*) If $f(2) = 4$, $f(2.5) = 5.5$, find the linear interpolating polynomial by using Lagrange interpolation and Newton's divided difference interpolation.