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NEPNY—16—2023

FACULTY OF SCIENCE

M.Sc. (NEP) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

PHYSICS

Paper SPHYC-401

(Mathematical Methods in Physics)

(Wednesday, 20-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—Three Hours *Maximum Marks*—80

N.B. :— (*i*) All questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Solve any *three* of the remaining *five* questions (Q. No. **2** to Q. No. **6**).

(*iv*) Figures to the right indicate full marks.

1. Solve the following questions : 20

(*a*) Rotation of a matrix

(*b*) Generating function of Bessel polynomial

(*c*) Fourier sine and cosine transform

(*d*) Cauchy residue theorem.

P.T.O.

2. (*a*) Find the eigen values, eigen vectors and diagonal matrix of the following :

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(*b*) Discuss Gram-schmidt's orthogonalization process in detail. 20

3. (*a*) Obtain the Rodrigue's formula of Legendre polynomial and find the values of :

$$P_0(x), P_1(x), P_2(x) \text{ and } P_3(x)$$

(*b*) Find the solution of Hermite polynomial i.e. :

$$y'' - 2xy' + 2ny = 0.$$ 20

4. (*a*) Define what is the Laplace transform and find the Laplace transform of.

(*i*) $f(t) = 1$

(*ii*) $f(t) = \sin at$

(*iii*) $f(t) = \cos at$

(*iv*) $f(t) = \sin hat$

(*v*) $f(t) = \cos hat$

(*b*) Explain the first and second shifting properties of Laplace transform and find the solution of differential equation using Laplace transform : 20

$$y'' + 25y = 10 \cos 5t,$$

where $y(0) = 2, \; y'(0) = 0.$

5. (*a*) Define analytic function and show that if $f(z)$ is analytic in and on the closed curve '*c*' and if '*a*' is any point on '*c*', then :

$$f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} dz.$$

(*b*) Evaluate $\int_c (x+y)dx + x^2 y dy$:

(*i*) Along $y = x^2$ having (0,0) and (3,9) as end points

(*ii*) Along $y = 3x$ between (0,0) and (3,9). 20

6. Write short notes on the following : 20

(*a*) Linear dependence and independence of vectors

(*b*) Rodrigue's formula of Laguerre's polynomial

(*c*) Properties and applications of Fourier series

(*d*) Singularities of an analytic function.