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NY—07—2023

FACULTY OF SCIENCE

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

PHYSICS

PHY–101

(Mathematical Methods in Physics)

(Tuesday, 05-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Each question carries equal marks.

(*iii*) Use of non-programmable calculator is allowed.

1. Define what is matrix and find the eigen values, eigen vectors and diagonal matrix of the following matrix : 15

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

P.T.O.

Or

(*a*) Find the eigen values and eigen vectors of the given matrix : 8

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(*b*) Describe Gram Schmidt's orthogonalization process. 7

2. Show that the generating function of Hermite polynomial is : 15

$$e^{2xt-t^2} = \sum_n \frac{Hn(x)t^n}{n!}$$

where $\frac{H_n(x)}{n!}$ is the coefficient of t^n in the expansion of e^{2xt-t^2} and prove that :

(*i*) $2xH_n(x) - H_{n+1}(x) = 2nH_{n-1}(x)$

(*ii*) $2nH_{n-1}(x) = H'_n(x)$.

Or

(*a*) Obtain the Rodrigues formula of $H_n(x)$. 8

(*b*) Show that : 7

(*i*) $(n + 1)\, P_{n+1}(x) = (2n + 1)x\, P_n(x) - nP_{n-1}(x)$

(*ii*) $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$.

3. Define Fourier series and find the Fourier series represented by the function : 15

$$F(x) = x, \quad 0 < x < 2\pi$$

Or

(*a*) If $f(s)$ is the Fourier transform of $f(x)$, then show that : 8

$$\mathrm{F}\left[f(x)\cos ax\right] = \frac{1}{2}\left[f(s+a) + f(s-a)\right].$$

(*b*) Using Laplace transform, find the solution of initial value problem : 7

$$y'' + 25y = 10 \cos 5t$$

$$y(0) = 2 \text{ and } y'(0) = 0.$$

4. Show that if $\mathrm{F}(z)$ is analytic in and on the closed curve 'c' and if 'a' is any point on 'c', then : 15

$$f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz \text{ and}$$

$$\text{evaluate } \int_C \frac{2z^3 + 3z + 5}{z-2}\,dz, \text{ where } c : |z| = 3.$$

Or

(*a*) Find the value of $\int_C (x+y)dx + x^2 y\,dy$: 8

(*i*) along $y = x^2$ having (0, 0) and (3, 9) as end points

(*ii*) along $y = 3x$ between (0, 0) and (3, 9).

(*b*) Show that the function :

$$u(x,\ y) = 2x - 2xy$$

is harmonic and find its conjugate harmonic function. 7

P.T.O.

5. Write short notes on (any *three*) : 15

(*a*) Symmetric and skew-symmetric matrix with suitable examples

(*b*) Cauchy Residue theorem

(*c*) Properties of Fourier transform

(*d*) Rodrigues formula of $P_n(x)$.