This question paper contains **4** printed pages]

NY—57—2023

FACULTY OF ARTS/SCIENCES

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–XIX

(Numerical Analysis)

(Wednesday, 06-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Scientific calculator is allowed.

1. Attempt the following : 15

(*a*) Define rate of convergence. Explain in detail Newton-Raphson method of solving the equation $f(x) = 0$. Prove that the Newton-Raphson method has quadratic rate of convergence.

Or

(*b*) Obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$ by using secant method. Perform rour iterations.

P.T.O.

2. Attempt the following : 15

(*a*) Define the following :

(*i*) Diagonally dominant matrix

(*ii*) Positive definite matrix.

Explain in detail Gauss elimination method of solving the system of equations $Ax = b$.

Or

(*b*) Solve the system of equations :

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$

by using LU decomposition method.

3. Attempt the following : 15

(*a*) (*i*) Discuss in detail Jacobi Iteration method of solving the system of equations $Ax = b$. Also obtain its error format.

(*ii*) State and prove Gerschgorin theorem.

Or

(*b*) Solve the system of equations :

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

by using Gauss-Seidel method. Take initial approximation as $x^{(0)} = 0$. Perform three iterations.

4. Attempt the following : 15

(*a*) Let function $f(x)$ be continuous on $[a, b]$ and its values are known at $n + 1$ distinct points $a \leq x_0 < x_1 < x_2 < ..... < x_{n-1} < x_n \leq b$ of $[a, b]$, then prove that there exists a unique polynomial $P(x)$ which satisfies the conditions $P(x_i) = f(x_i) \quad \forall\, i = 0, 1, 2 ..... n$ if Vandermonde's determinant is non-zero.

Or

(*b*) Find the unique polynomial of degree two or less, such that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$ by using :

(*i*) Lagrange interpolation

(*ii*) Iterated interpolation

(*iii*) The Newton's Divided Difference Interpolation.

P.T.O.

5. Attempt any *three* of the following : 5 marks each

(*a*) Perform two iterations of the Chebyshev method to obtain the approximate value of $\frac{1}{7}$. Take the initial approximation as $x_0 = 0.1$.

(*b*) Calculate the nth divided difference of $\frac{1}{x}$, based on the points $x_0, x_1, \ldots\ldots, x_n$.

(*c*) Determine the inverse of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

using the partition method.

(*d*) Find the interval which contains the eigenvalues of the symmetric matrix :

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$