This question paper contains **4** printed pages]

NY—227—2023

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–XXI–C

(Fuzzy Sets and Their Applications–II)

(Monday, 11-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define a fuzzy measure function in detail and explain axioms boundary conditions, monotonicity, continuity with statement. 15

Or

Given a consonant body of evidence (**H**, *m*), then prove that the associated consonant belief and plausibility measures possess the following property :

(*i*) $\text{Bel}\ (A \cap B) = \min\ [\text{Bel}\ (A), \text{Bel}\ (B)]$ for all $A, B \in \mathbf{P}(X)$

(*ii*) $\text{Pl}\ (A \cup B) = \max\ [\text{Pl}\ (A), \text{Pl}\ (B)]$ for all $A, B \in \mathbf{P}(X)$.

P.T.O.

2. Define measure of fuzziness, degree of fuzziness and the term maximally fuzzy in *f*(A) assumes the maximum value if and only if A is maximally fuzzy, by the membership grade $\frac{1}{2}$ for all $x \in X$. 15

Or

Prove that function $I(N) = \log_2 N$ is the only function that satisfies :

Axiom I1 : $I(N.M) = I(N) + I(M)$ for all N, $M \in \mathbf{N}$

Axiom I2 : $I(N) \leq I(N + 1)$ for all $N \in \mathbf{N}$

Axiom I3 : $I(2) = 1$

3. Define the write the formulae for dissonance in evidence, confusion in evidence, non-specificity in evidence, measure of fuzziness with its formula. 15

Or

Consider a universal set X, three non-empty subsets of which are of our interest : A, B and $A \cap B$. Assume that the only evidence is expressed in terms of the total belief focusing on A and the total belief focusing on B. The aim is to estimate the basic assignment values for X, A, B and $A \cap B$. The use of the principle of maximum non-specificity leads in this case to the following optimization problem.

Determine values $m(X)$, $m(A)$, $m(B)$ and $m(A \cap B)$ for which the function $m(X)\log_2 |X| + m(A)\log_2 |A| + m(B)\log_2 |B| + m(A \cap B)\log_2 |A \cap B|$ reaches its maximum subject to the constraints.

$m(A) + m(A \cap B) = a$

$m(B) + m(A \cap B) = b$

$m(X) + m(A) + m(B) + m(A \cap B) = 1$

$m(X), m(A), m(B), m(A \cap B) \geq 0$

where $a, b \in [0, 1]$ are given numbers.

4. Given a family of finite sets $X_1, X_2, \ldots\ldots, X_n$ and marginal probability distributions ${}^1P, {}^2P, \ldots\ldots\ldots\ldots\; {}^sP$ defined on some lower-dimensional cartesian products of $X_1 \times X_2 \times X_3 \times \ldots\ldots\ldots\ldots\; X_n = X$ that cover all sets in the family and represent a loopless structure system, the joint probability distribution determined by $p(x_1, x_2, \ldots\ldots, x_n) = \prod_{j=1}^{s} j_p$ is characterized by the maximum value of Shannon entropy among all joint distributions verify that satisfy the given marginal distributions. 15

Or

Give the difference between crisp and fuzzy data for two variables each with three states (values, labels) 0, 1, 2. Define pseudo-frequencies and derive expression for pseudo-frequencies.

P.T.O.

5. Attempt any *three* out of four : 15

(*a*) Write a short note on fuzzy database model and its associated fuzzy relational algebra with suitable example.

(*b*) Write a short note on 'Hartley information'.

(*c*) Prove that $H(X|Y) = H(X, Y) - H(Y)$

(*d*) Derive the formula for two-conditional probability distributions $P_{X|Y}$ and $P_{Y|X}$ and prove that the sets X and Y are independent of each other if and only if they are non-interactive.