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NY—225—2023

FACULTY OF ARTS / SCIENCE

M.A./M.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper XXI

(Classical Mechanics)

(Monday, 11-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—Three Hours *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Derive the Lagrange's equation of motion from D'Almbert's principle. Also, obtain an equation of motion in linear harmonic oscillator. 15

Or

(*b*) Define the following : 15

(*i*) Scleronomic constraint

(*ii*) Rheonomic constraint

and give *one* example of each.

Also, find the equation of motion for a bead sliding on uniformly rotating wire in force free space.

P.T.O.

2. (*a*) Derive Hamiltonian canonical equation from variational principle. Also, prove that the generalised momentum conjugate to a cyclic co-ordinate is conserved. 15

Or

(*b*) Define Routh's function and obtain the Routh's equation of motion. Also, find the Hamiltonian of simple pendulum. 15

3. (*a*) Prove that the necessary condition for : 15

$$J[y(x)] = \int_{x_1}^{x_2} F(x, y, y)\, dx$$

to be extremum is :

$$F_y - \frac{d}{dx} F_{y'} = 0 .$$

Or

(*b*) State and prove the fundamental lemma of calculus of variation. Find the extremal of the functional : 15

$$J[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\, dx, \text{ with } y(0) = 0, \quad y(\pi/2) = 1 .$$

4. (*a*) Discuss the invariance of Euler's equation. Explain Brachistochrone problem and find its extremal. 15

Or

(*b*) Define Geodesic and that the geodesic on the surface of a sphere is an arc of the great circle. 15

5. Attempt any *three* of the following : 5 each

(*a*) Find the equation of kinetic energy of rotating rigid body.

(*b*) If the Lagrangian L is given by :

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - \frac{k}{2}(x^2 - y^2) ,$$

then find its equation of motion in Hamiltonian formulation.

(*c*) Solve :

$$J[y(x)] = \int_a^b \frac{y^2}{x^3} .$$

(*d*) Discuss the case, the functional F is independent of x.