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NY—98—2023

FACULTY OF ARTS & SCIENCE

M.A./M.Sc. (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper : XIV

(Topology)

(Thursday, 07-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (1) *All* questions are compulsory.

(2) Figures to the right indicate full marks.

1. (*a*) If **B** is a basis for the topology of X, then the collection : 15

$$\mathbf{B}_y = \{B \cap Y / B \in \mathbf{B}\}$$

is a basis for the subspace topology on Y. Moreover, as Y be a subspace of X and U is open in Y with Y is open in X, then prove that U is open in X.

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Or

(*b*) Let X be a topological space. Then the following conditions hold :

(1) ϕ and X are closed.

(2) Arbitrary intersections of closed sets are closed. 15

2. (*a*) Let X and Y be topological spaces. 15

Let $f : X \to Y$, then the following are equivalent :

(1) f is continuous.

(2) For every subset A of X, one has $f\left(\overline{A}\right) \subset \left(\overline{f(A)}\right)$.

(3) For every closed set B of Y, the set f^{-1} (B) is closed in X.

Or

(*b*) State and prove maps into products theorem also prove that, every constant function from any topological space (X, J) to (Y, J') is continuous.

15

3. (*a*) State and prove Extreme Value Theorem : 15

Or

(*b*) Let X be a Hausdorff space. Then X is locally compact iff given x in X and given a neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$. 15

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4. (*a*) A subspace of a Hausdorff space is Hausdorff, a product of Hausdorff space is Hausdorff. Also prove that, subspace of a regular space is regular. 15

Or

(*b*) If X has a countable basis, then prove that every open covering of X contains a countable subcollection covering X. Also prove that, subspace of a second countable space is second-countable.

5. Attempt any *three* of the following : 15

(*a*) If A is closed in Y and Y is closed in X, then prove that A is closed in X.

(*b*) Prove that, the function $f : R \to R$ given by $f(x) = nx^2 + 1$ is needn't be homomorphism. 15

(*c*) Prove that, product of second countable space is second countable.

(*d*) Prove that, every compactness implies limit point compactness.

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