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NY—183—2023

FACULTY OF SCIENCE AND TECHNOLOGY

M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–XV–C

(Fuzzy Sets and Their Applications–I)

(Saturday, 9-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define fuzzy set with suitable examples and explain why we need fuzzy set theory. Whether the law of contradiction and the law of exclusive middle are valid in fuzzy set theory ? Justify your claim. 15

Or

Define in the form of a table primitives $\wedge, \vee, \Rightarrow$ and $\Leftrightarrow$, of the Tukasiewicz, Bochvar logics L_3 and explain the way by which we can extend two-valued logic into three-valued logic.

P.T.O.

2. Write the set of axioms of the axiomatic skeleton for fuzzy set unions and prove that : 15

$$\lim_{w \to \infty} \min\left[1, (a^w + b^w)^{1/w}\right] = \max(a, b)$$

Or

Prove that the following :

(*i*) For all $a, b \in [0, 1]$, $u(a, b) \geq \max(a, b)$

(*ii*) for all $a, b \in [0, 1]$, $u(a, b) \leq u_{\max}(a, b)$.

3. Explain the concept of fuzzy relation, kinds of fuzzy relation and fuzzy matrix with their suitable examples. 15

Or

Consider the sets $X_1 = \{x, y\}$, $X_2 = \{a, b\}$ and $X_3 = \{*, \$\}$ and the ternary fuzzy relation :

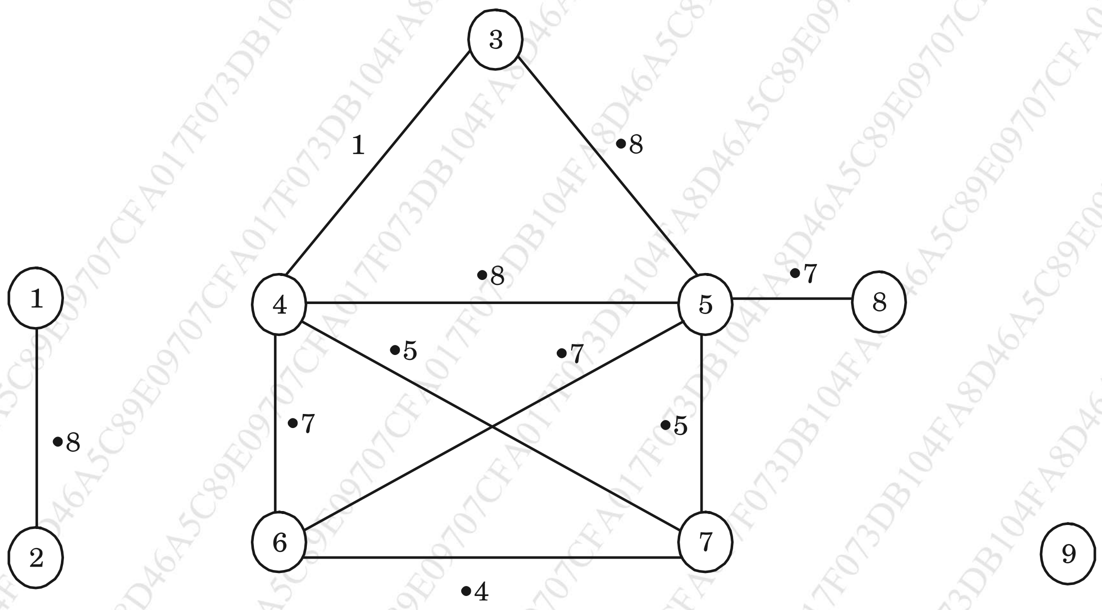
$R(X_1, X_2, X_3) = 0.9/x, a, * + 0.4/x, b, * + 1/y, a, *, + 0.7/y, a, \$ + 0.8/y, b, \$$

defined on $X_1 \times X_2 \times X_3$. Let

$$R_{i,j} = \left[R \downarrow \{X_i, X_j\}\right] \text{ and } R_i = \left[R \downarrow \{X_i\}\right]$$

for all $i, j \in \mathbf{N}_3$, then calculate $R_{1,2}$, $R_{1,3}$, $R_{2,3}$, R_1, R_2, R_3 in the form of table.

4. Define compability relation in fuzzy set and from the following graph represent membership matrix for a fuzzy relation R(X, X) defined on X = $\mathbf{N}_9$ and depict its complete α–covers for α > 0 $\alpha \in \Lambda_R = \{0, 0.4, 0.5, 0.7, 0.8, 1\}$: 15



Or

Define morphisms in fuzzy relation. Explain ordinary fuzzy homomorphism and strong hormomorphism with suitable examples and draw pictorial diagram.

5. Attempt any *three* out of four : 15

(*a*) Consider the matrix equation

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \bullet \begin{bmatrix} \cdot 9 & \cdot 5 \\ \cdot 7 & \cdot 8 \\ 1 & \cdot 4 \end{bmatrix} = \begin{bmatrix} \cdot 6 & \cdot 3 \\ \cdot 2 & 1 \end{bmatrix}$$

P.T.O.

whose general form is

$[P_{i,j}]\ [q_{j,k}] = [r_{i,k}]$, where $i \in \mathbf{N}_2$, $j \in \mathbf{N}_3$ and $k \in \mathbf{N}_2$. Show that the given matrix has no solution.

(*b*) Represent the following fuzzy relation R(X, X) in simple and sagittal diagram :

X	Y	$\mu_R(X, Y)$
1	1	.7
1	3	.3
2	2	.7
2	3	1
3	1	.9
3	4	1
4	3	.8
4	4	.5

(*c*) Prove that every fuzzy complement has atmost one equilibrium.

(*d*) Compute the scalar cardinality for each of the following fuzzy sets :

(*i*) $A = 0.4/v + 0.2/w + 0.5/x + 0.4/y + 1/z$

(*ii*) $B = 1/x + 1/y + 1/z$.