This question paper contains **3** printed pages]

NY—181—2023

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–XV(A)

(Analytical Number Theory)

(Saturday, 9-12-2023) **Time : 2.00 p.m. to 5.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. State and prove the Chinese remainder theorem. Also show that 41 divides $2^{20} - 1$. 15

Or

State and prove Wilson theorem. Also, solve the linear congruence $18x \equiv 30 \pmod{42}$.

2. Let the integer a have order k modulo n, then prove that $a^h \equiv 1 \pmod{n}$ if and only if $k \mid h$. Find two primitive roots of 10. 15

P.T.O.

Or

For $k \geq 3$, prove that 2^k has no primitive roots. Also, verify that, 2 is a primitive root of 19, but not of 19.

3. Let p be an odd prime and gcd $(a, p) = 1$, then prove that a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1$ (mod p). Also, find the values of the following Legendre symbols : 15

(*i*) (19/23)

(*ii*) (–23/59).

Or

State and prove quadratic reciprocity law. Use Gauss-Lemma to compute each of Legendre symbols : 15

(*i*) (8/11)

(*ii*) (5/19).

4. If $n \geq 1$, then prove that : 15

$$\sum_{d/n} \mu(d) = \left[\frac{1}{n}\right] = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1 \end{cases}$$

Also, define Mangoldt function $\Lambda(n)$. Find the values of Mangoldt function for :

$n = 1$ to 10.

Or

State and prove Selberg identity. Also, find :

(*i*) $\phi(343)$

(*ii*) $\phi(4300)$.

5. Attempt any *three* of the following : 5 each

(*a*) Give an example to show $a^2 \equiv b^2 \pmod{n}$ does not implies $a \equiv b \pmod{n}$.

(*b*) Verify that $2^{340} \equiv 1 \pmod{341}$.

(*c*) Write a short note on a Pythagorean triple with suitable examples.

(*d*) Find all integers n such that $\phi(n) = \phi(2n)$.