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NEPNY—81—2023

FACULTY OF SCIENCE/ARTS

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

MATHEMATICS

Paper–SMATE–401 (A)

(Ordinary Differential Equations)

(Thursday, 28-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Solve $3y'' + 2y' = 0$

(*b*) Verify that $\phi_1(x) = x^3$ $(x > 0)$ satisfies the equation $x^2y'' - 7xy' + 15y = 0$ and find the second independent solution.

(*c*) Solve $x^2y'' + 2xy' - 6y = 0$ for $x > 0$.

(*d*) Solve $f(x, y) = xy^2$ on $R : |x| \le 1, |y| \le 1,$

P.T.O.

2. Answer the following : 20

(*a*) Prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$, where ϕ_1, ϕ_2 are two linearly independent solutions of $L(y) = 0$ on an interval I containing a point x_0.

(*b*) Solve $4y'' - y = e^x$.

3. Answer the following : 20

(*a*) Derive *two* solutions of Legendre equation

(*b*) Solve $y'' - xy = 0$ by using power series expansion.

4. Answer the following : 20

(*a*) Find *two* solutions of Euler's equation.

(*b*) Prove that the solution of $x^2y'' + a(x)\, xy' + b(x)y = 0$ is given by $\phi_1(x) = |x|^{r_1} \sum_{k=0}^{\infty} C_k x^k$, where $i = 1, 2$ and a, b have convergent power series expansions.

5. Answer the following : 20

(*a*) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I if and only if it is solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$ on 1.

(*b*) Prove that M (x, y) + N $(x, y)\, y' = 0$ is exact on some rectangle R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

6. Answer the following : 20

(*a*) Solve $y''' - y' = x$.

(*b*) Prove that the solution of $y' = xy,\ y(0) = 1$ is $e^{\frac{x^2}{2}}$.