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**NEPNY—83—2023**

**FACULTY OF SCIENCE**

**M.Sc. (NEP) (First Year) (First Semester) EXAMINATION**

**NOVEMBER/DECEMBER, 2023**

**MATHEMATICS**

**Paper—SMATE—401 (C)**

**(Dynamics and Continuum Mechanics—I)**

**(Thursday, 28-12-2023)**

**Time : 10.00 a.m. to 1.00 p.m.**

*Time—3 Hours*

*Maximum Marks—80*

**N.B. :-** (i) All questions carry equal marks.

(ii) Q. No. 1 is compulsory.

(iii) Answer any *three* from Q. No. 2 to Q. No. 6.

(iv) Figures to the right indicate full marks.

1. Attempt the following (5 marks each) : 20

(a) If  $r = [x, y, z]$ , prove that (i)  $\text{div } r = 3$ , (ii)  $\text{curl } r = 0$ .

(b) Prove that, gravitational force is conservative.

(c) Define (i) Newton's laws of Motion, (ii) Force.

(d) Find an equimomental system of particles for a uniform rod AB of mass M, where O be the centroid of the rod,  $2a$  its length.

P.T.O.

2. Attempt the following (10 marks each) : 20

(a) Define vector couple and show that a centroid of the system is unique.

(b) Show that :  $\nabla (\phi_1 \cdot \phi_2) = \phi_1 \nabla \phi_2 + \phi_2 \nabla \phi_1$ .

3. Attempt the following (10 marks each) : 20

(a) State and prove the principle of conservation of energy.

(b) A smooth wire bent in the form of parabola is fixed with its axis vertical and vertex downwards. A particle of mass  $m$  oscillate on the wire coming to rest at extremities of the latus rectum. Show that the reaction of the wire on the particle when passing through the vertex is  $2 mg$ .

4. Attempt the following (10 marks each) : 20

(a) Prove the parallel axis for moment of inertia and product of inertia.

(b) Two particle of masses  $m_1$  and  $m_2$  at A and B connected by a rigid massless rod AB, their velocities are  $\vec{v}_1$  and  $\vec{v}_2$  are suddenly changed by the application of externally impulses  $\vec{J}_1$  and  $\vec{J}_2$ . Prove that the magnitude  $f$  of impulsive reaction of the rod on  $m_1$  is

$$\frac{m_1 \cdot m_2}{m_1 + m_2} e \left[ \frac{\vec{J}_1}{m_1} - \frac{\vec{J}_2}{m_2} \right].$$

5. Attempt the following (10 marks each) : 20

(a) Prove that kinetic energy of rigid body in a general motion is :

$$T = \frac{1}{2} M \bar{v}^2 + \frac{1}{2} [A^* \omega_1^2 + B^* \omega_2^2 + C^* \omega_3^2]$$

(b) With the usual notations, prove that  $\tan 2\alpha = \left( \frac{2F}{B - A} \right)$ .

6. Attempt the following (10 marks each) : 20

(a) Show that there is no loss of energy for a perfectly elastic impact.

(b) Determine the moment of inertia of the distribution about the axis through O having direction cosines  $[\lambda, \mu, \nu]$  in terms of these direction cosines and A, B, C, D, E, F.