This question paper contains **3** printed pages]

NEPNY—50—2023

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

MATHEMATICS

Paper–SMAT–C403

(COMPLEX ANALYSIS)

(Tuesday, 26-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Q. No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Find all the values of z such that $e^z = 5 + 5i$.

(*b*) Find the length of the curve C : $z(t) = 3e^{2it} + 2$ $(-\pi \leq t \leq \pi)$.

(*c*) $\int_C \frac{e^{z^2}}{(z-2)}\, dz$ where, C : $|z| = 3$.

(*d*) Find the principal part of the Laurent's expansion for the function $f(z) = \frac{z}{z^2+4}$ valid in the neighborhood of $z = 2i$.

P.T.O.

2. Answer the following : 20

(*a*) Prove that, for given three distinct points z_1, z_2 and z_3 in extended z-plane and three distinct points w_1, w_2 and w_3 in extended w-plane there exist a unique bilinear transformation $w = T(z)$ such that $T(z_k) = w_k$ for

$$k = 1, 2, 3.$$

(*b*) Show that the exponential function $f(z) = e^z$ is periodic function with purely imaginary period $2\pi i$. Also show the following :

(*i*) $\sin 2z = 2 \sin z . \cos z$ (*ii*) $\sin\left(\frac{\pi}{2} + z\right) = \cos z$.

3. Answer the following : 20

(*a*) Define Contour. State and prove Cauchy's main theorem.

(*b*) Find the values of a, b and c such that the following functions,

(*i*) $f(z) = a(x^2 + y^2) + ibxy + c$ (*ii*) $f(z) = x + ay - i(bx + cy)$ are entire.

4. Answer the following : 20

(*a*) State and prove Cauchy's Integral Formula.

(*b*) Evaluate

(*i*) $\int_C \frac{3z^4 + 2z - 6}{(z-2)^3} dz$ where, C : $|z| = 3$.

(*ii*) $\int_C \frac{z - 3\cos z}{\left(z - \frac{\pi}{2}\right)^2} dz$ where, C : $|z| = 2$.

5. Answer the following : 20

(*a*) State and prove Casorati-Weierstrass' Theorem.

(*b*) (*i*) Find [Res : $f(z)$; $z = 1$] for the function $f(z) = \dfrac{z^4 - z^3 + 17z + 12}{(z-1)^3}$.

Also evaluate $\int_C f(z)\, dz$.

(*ii*) Evaluate $\int_C \dfrac{1}{z(z-3)} dz$ along any simple closed contour C.

6. Answer the following : 20

(*a*) Define length of the curve. State and prove M-L inequality theorem.

(*b*) Find all the singularities of the function $f(z) = \cot \pi z$. Also find the principal part of Laurent's expansion in the deleted neighborhood of the each singularity.