This question paper contains **3** printed pages]

NEPNY—14—2023

FACULTY OF SCIENCE AND TECHNOLOGY

M.A./M.Sc. (NEP) (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

MATHEMATICS

Paper–SMATC–401

(Algebra)

(Wednesday, 20-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—80

N.B. :— (*i*) *All* questions carry equal marks.

(*ii*) Question No. **1** is compulsory.

(*iii*) Answer any *three* from Q. No. **2** to Q. No. **6**.

(*iv*) Figures to the right indicate full marks.

1. Answer the following : 20

(*a*) Prove that every finite group has a composition series.

(*b*) Prove that there is no simple group of order 63.

(*c*) Prove that intersection of ideals is a ring R is again ideal.

P.T.O.

(*d*) Prove that a group homomorphism $\phi : G \to H$ is injective if and only if ker $(\phi) = \{e\}$.

2. Answer the following : 20

(*a*) State and prove first isomorphism theorem.

(*b*) Prove that every infinite cyclic group is isomorphic to **Z**.

3. Answer the following : 20

(*a*) Write down all the composition series for the Q_8.

(*b*) If G be a nilpotent group, then every subgroup of G and every homomorphic image of G are nilpotent.

4. Answer the following : 20

(*a*) Find the non-isomorphic abelian group of order 16.

(*b*) State and prove first sylow theorem.

5. Answer the following : 20

(*a*) Prove that every Euclidian domain is a UFD.

(*b*) Prove that an ideal M in the ring of integer Z is maximal ideal if and only if M = <P>, P is prime number.

6. Answer the following : 20

(*a*) Prove that a sylow P-subgroup of a finite group G is unique if and only if it is normal.

(*b*) If H and K are normal subgroup of G and $K \subset H$, then prove that $(G/K)/H/K \quad G/H$.