NY—222—2023

FACULTY OF SCIENCE

M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–IX

(Partial Differential Equation)

(Monday, 11-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 Hours *Maximum Marks—75*

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Find the general integral of the following : 15

(*i*) $xp + yq = z$

(*ii*) $yxp + xzq = xy$

(*iii*) $z(xp - yq) = y^2 - x^2$.

Or

(*b*) The necessary and sufficient condition that Pfaffian differential equation $\vec{X}\,d\vec{r} = P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz$ is integrable that $\vec{X}.\mathrm{curl}\left(\vec{X}\right) = 0$.

P.T.O.

2. (a) Find the complete integral of the following by Charpit's method : 15

(i) $F = z^2 - pqxy = 0$

(ii) $F = xpq + yq^2 = 0$.

Or

(b) Show that the equations

$f = xp - yq - x = 0$ and $g = x^2p + q - xz = 0$ are compatible and find a parameter family of the common solution. 15

3. (a) Find the complete integral of the equation $p^2x + q^2y = z$ by Jacobian's method. 15

Or

(b) Find a complete integral of the equation $(p^2 + q^2)x = pz$ the integral surface containing the curve $x_0 = 0$, $y_0 = s^2$, $z_0 = 25$. 15

4. (a) Derive an expression for vibration of a semilinear string. 15

Or

(b) State and prove Harnack's theorem. 15

5. Attempt any *three* of the following : 15

(a) Find the general solution of $xp + yq = z$.

(b) Find complete integral of $xpq + yq^2 - 1 = 0$.

(c) Find the equation of surface satisfying $4\,yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1$ and $x + z = 2$.

(d) State and prove the necessary condition for Neumann problem.