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NY—386—2023

FACULTY OF SCIENCE

M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper-XI (B)

(Dynamics and Continuum Mechanics-II)

(Friday, 15-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

Note :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt the following :

(*a*) Define a tensor and find its components. 8

(*b*) A rigid body is rotated clockwise through 90° about $\hat{e}_3$ axis then prove that this tensor $\bar{R}$ is orthogonal. Verify that $[\bar{R}][\bar{R}]^T = [\bar{I}]$, also find det $[\bar{R}]$. 7

P.T.O.

Or

(*c*) Prove that matrix of tensor with respect to principal direction is diagonal and also prove that the principal value of tensor $\bar{T}$ include the maximum value that the diagonal elements of any matrix of $\bar{T}$ we have,

$$\frac{\Delta(d\bar{v})}{d\bar{v}} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 \qquad 8$$

(*d*) Define dyadic product of two vectors $\bar{a}$ and $\bar{b}$ and prove that dyadic product of $\bar{a}$ and $\bar{b}$ is a tensor and obtain its components. 7

2. Attempt the following :

(*a*) Obtain six equations of compatibility for infinitesimal strain components. 8

(*b*) For a plane stress find the principal values and corresponding principal directions. 7

Or

(*c*) Define transpose of a tensor and with usual notations show that :

$$[\bar{Q}][\bar{Q}^T] = [\bar{Q}^T][\bar{Q}] = [\bar{I}] \qquad 8$$

(*d*) Define eigen values and eigen vector of tensor and explain how are these determined. 7

3. Attempt the following :

(*a*) Derive Cauchy's equation of motion. 8

(*b*) Show that there is S_{ij} are Cartesian components of a tensor $\bar{S}$, then $S_{ii} = S_{11} + S_{22} + S_{33}$ is a scalar invarient with respect to all orthogonal. 7

Or

(*c*) Derive an expression for Cartesian co-ordinates of rate of deformation and spin tensor. 8

(*d*) Prove that $\frac{\partial T_{ij}}{\partial_{xj}} + P_{Bj} = P_{ai}$. 7

4. Attempt the following :

(*a*) The motion of a body is :

$x_1 = x_1 + kt\ x_2,\ x_2 = x_2,\ x_3 = x_3$

if the temperature field is given by the spatial description $\theta = x_1 + x_2$ obtain velocity and rate of change of temperature for particular material particles. 8

(*b*) Define and explain Dilation. 7

Or

(*c*) Discuss divergence of a vector field, divergence of a tensor field and curls of a vector field. 8

(*d*) Define strain tensor. Write its components. 7

P.T.O.

5. Attempt any *three* of the following : 15

(*a*) Find velocity field associated with motion of rigid body relation with angular velocity :

(*i*) $\bar{w}=w\hat{e}_3$

(*ii*) Using the velocity field of part (*i*) evaluate acceleration field.

(*b*) State and prove equations of hydrostatics.

(*c*) Explain gradient of vector field.

(*d*) Obtain equation of conservation of mass.

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