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NY—385—2023

FACULTY OF ARTS/SCIENCE

M.A./M.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper IX(A)

(Combinatorics)

(Friday, 15-12-2023)

Time : 10.00 a.m. to 1.00 p.m.

Time—Three Hours

Maximum Marks—75

N.B. :— (i) All questions are compulsory.

(ii) Figures to the right indicate full marks.

1. Attempt the following :

15

- (a) How many arrangements of the seven letters of the word SYSTEMS can have the E occurring somewhere before the M ? How many arrangements have the E somewhere before M and the three Ss grouped consecutively ?

What is the probability that a 4-digit campus telephone number has one or more repeated digits ?

Or

- (b) Nine students, three from Ms. A's class, three from Mr. B's class, and three from Ms. C's class have bought a block of nine seats for their school's homecoming game. If three seats are randomly selected for each class from the nine seats in a row, what is the probability that the three A students, three B students, and three C students will each get a block of three consecutive seats ?

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2. Attempt the following :

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- (a) Use a generating function to model the problem of counting all selections of six objects chosen from three types of objects with repetition of up to four objects of each type. Also model the problem with unlimited repetition.

Or

- (b) How many ways are there to distribute 25 identical balls into 7 distinct boxes if first box can have no more than 10 balls but any number can go into each of the other six boxes ?

3. Attempt the following :

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- (a) A bank pays 4 percent interest each year on money on savings account. Find recurrence relations for the amount of money a gnome would have after n years if it follows the investment strategies of :
- (i) Investing Rs. 1,000 and leaving it in the bank for n years.
- (ii) Investing Rs. 100 at the end of each year.

Or

- (b) An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for a_n the number of different ways for the elf to ascend the n -stair staircase.

4. Attempt the following :

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- (a) Let A_1, A_2, \dots, A_n be n sets in the universal set U . Then with usual notations, prove that :

$$N(A_1 \cup A_2 \cup \dots \cup A_n) = S_1 - S_2 + S_3 - \dots + (-1)^{k-1} S_k + \dots + (-1)^{n-1} S_n.$$

- (b) Find the number of 4-digit ternary sequences with exactly two 1s. Also, find the number with at least two 1s.

5. Attempt any *three* of the following :

15

- (a) How many arrangements are there of the six letters, b, a, n, a, n, a ?
- (b) Find a generating function for a_r , the number of ways to distribute r identical objects into five distinct boxes with an even number of objects not exceeding 10 in the first two boxes and between three and five in the other boxes.
- (c) Suppose we draw n straight lines on a piece of paper so that every pair of lines intersect (but no three lines intersect at a common point). Into how many regions do these n lines divide the plane ?
- (d) How many different integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20, 0 \leq x_i \leq 8$?