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NY—176—2023

FACULTY OF SCIENCE

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS Pattern)

MATHEMATICS

Paper–III

(Ordinary Differential Equation)

(Saturday, 09-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.L. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attempt any *one* of the following :

(*a*) ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I. Prove that every solution ϕ of $L(y) = 0$ can be written uniquely as $\phi = C_1\phi_1 + C_2\phi_2$ 15

Where C_1, C_2 are constants.

(*b*) Prove that every solution ψ of

$L(y) = b(x)$ is $\psi = \psi_P + C_1\phi_1 + C_2\phi_2 + ...+ C_n\phi_n$. 15

Where ψ_P is particular solution and b be continuous on an interval I.

P.T.O.

2. Attempt any *one* of the following : 15

(*a*) Prove that the second solution of $y'' + a_1(x)y' + a_2(x)y = 0$ is $\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{\phi_1(5)^2} \exp\left[-\int_{x_0}^{x} a_1(t)\, dt\right] ds$.

(*b*) Solve $y'' - xy = 0$ by using power series expansion.

3. Attempt any *one* of the following : 15

(*a*) Solve $x^2y'' + axy' + by = 0$.

(*b*) Solve Bessel equation to find Bessel function of zero order of the first kind.

4. Attempt any *one* of the following : 15

(*a*) Prove that $M(x, y) + N(x, y)y' = 0$ is exact in R : $|x - x_0| \leq a$, $|y - y_0| \leq b$ if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

(*b*) Prove that ϕ is a solution of $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I if and only if it is solution of $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$ on I.

5. Attempt any *three* of the following : 15

(*i*) Solve : $y'' - y' - 2y = e^{-x}$

(*ii*) Find the second solution of

$$y'' - \frac{2}{x^2}y = 0 \ (0 < x < \infty).$$

First solution is $\phi_1(x) = x^2$.

(*iii*) Express whether the singular point is regular singular point for

$$x^2y'' - y' - \frac{3}{4}y = 0.$$

(*iv*) Solve $y' = xy$, $y(0) = 1$.