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NY—15—2023

FACULTY OF ARTS AND SCIENCE

M.A./M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New/CBCS)

MATHEMATICS

Paper-I

[Abstract Algebra-I (Group & Ring Theory)]

(Tuesday, 05-12-2023) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Prove that, every cyclic group of order n is isomorphic to Z_n. Also prove that, every subgroup of cyclic group is cyclic. 15

Or

(*b*) State and prove second isomorphism theorem. 15

2. (*a*) Prove that, subgroup of solvable group is solvable. Also, prove that homomorphic image of solvable group is solvable. 15

Or

(*b*) The set Aut (G) of all automorphism of a group G is a group under composition of mappings, and In (G), Δ Aut (G). Moreover

P.T.O.

G/Z(G) $\approx$ In (G). 15

3. (*a*) Prove that, a sylow p-subgroup of a finite group G is unique iff it is normal. Also, prove that there are no simple group of order 56. 15

Or

(*b*) State and prove third Sylow theorem. Also, find the non-isomorphic abeian groups of order $360 = 2^3.3^2.5^1$. 15

4. (*a*) State and prove fundamental theorem of homomorphisms. 15

Or

(*b*) In a nonzero commutative ring with unity, an ideal M is maximal iff R/M is a field. 15

5. Attempt any *three* of the following : 15

(*a*) If H and K are cyclic group of order m & n respectively such that $(m, n) = 1$. Then $H \times K$ is a cyclic group order $m.n$.

(*b*) An abelian group G has a composition series iff G is finite.

(*c*) Let $f : R \rightarrow S$ be a Homomorphism of a ring R into a ring S. Then Ker $f = (0)$ iff f is 1–1.

(*d*) If G is a group of order pq, where p and q are distinct primes and if a has a normal subgroup H of order p and a normal subgroup K of order q, then prove that G is cyclic..