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NA—26—2023

FACULTY OF SCIENCE

B.Sc. (VI Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper XV

(Complex Analysis)

(Friday, 8-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Explain the method to find the nth roots of non-zero complex number z_0. Hence calculate the cube root of $(-8i)$. 15

Or

(*a*) Define harmonic function. Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component function u and v are harmonic in D. 8

(*b*) Show that $u(x, y) = y^3 - 3x^2y$ is harmonic in some domain and find harmonic conjugate $v(x, y)$. 7

P.T.O.

2. Let f be analytic function everywhere inside and on a simile closed contour C, taken in the positive sense. If z_0 is any interior point to C, then prove that : 15

$$f'(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)\, dz}{z - z_0}$$

Hence evaluate $\int_C \frac{z\,dz}{2z+1}$.

Or

(*a*) If $w(t)$ is a piecewise continuous complex-valued function defined on an interval $a \le t \le b$, then prove that : 8

$$\left| \int_a^b w(t)\, dt \right| \le \int_a^b |w(t)|\, dt.$$

(*b*) Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant, then show that : 7

$$\left| \int_C \frac{z+4}{z^3 - 1}\, dz \right| \le \frac{6\pi}{7}.$$

3. Attempt any *two* of the following :

(*a*) Suppose that a function f is analytic inside and on a positively oriented circle C_R, centered at z_0 and within radius R. If M_R denotes the maximum value of $|f(z)|$ on C_R, then prove that : 5

$$|f^{(n)}(z_0)| \le \frac{n!\, M_R}{R^n}$$

$(n = 1, 2, \ldots\ldots)$.

(*b*) Find the value of the integral : 5

$$\int_C \bar{z}\, dz .$$

(*c*) Find the numbers $z = x + iy$ such that : 5

$$e^z = 1 + i \cdot$$

(*d*) If $f(z) = \frac{i\bar{z}}{2}$ in the open disc $|z| < 1$, then show that : 5

$$\lim_{z \to 1} f(z) = \frac{i}{2} .$$