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NA—78—2023

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Part-XIV

(Operation Research)

(Saturday, 16-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. What are the basic assumptions necessary for all linear programming problems ? 15

Also, formulate the problem for maximising the profit.

The manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs are as follows :

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amount available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline

P.T.O

X and 130 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 4 and Rs. 5 respectively.

Or

(*a*) Using graphical method solve the following LPP :

Minimize $Z = -x_1 + 2x_2$; 8

Subject to constraints

$$-x_1 + 3x_2 \leq 10,$$
$$x_1 + x_2 \leq 6$$
$$x_1 - x_2 \leq 2$$

and $x_1, x_2 \geq 0$

(*b*) Define slack and surplus variables. Also, explain the canonical form for general formulation of linear programming problem. 7

2. A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below (ijth entry) : 15

$$\text{Cost matrix} : \begin{pmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{pmatrix}$$

Then, draw the associated network. Formulate the network LPP and find the minimum cost of making the assignment.

Or

(*a*) Obtain the inverse of matrix using simplex method $A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$. 8

(*b*) Define the degenerate solution and show that the following system of linear equations has a degenerate solution : 7

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

3. Attempt any *two* questions from the following :

(*a*) What are the *three* components of a linear programming problem ? Explain them. 5

(*b*) Prove that, the set of feasible solutions to an LPP is a convex set. 5

(*c*) Let $x_1 = 2$, $x_2 = 4$ and $x_3 = 1$ be a feasible solution to the system of equations. 5

$$2x_1 - x_2 + x_3 = 2$$
$$x_1 + 4x_2 = 18$$

Then, reduce the given feasible solution to a basic feasible solution.

(*d*) Prove that, a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that :

Total supply = Total demand. 5