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NA—80—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper XIV

(Numerical Analysis)

(Saturday, 16-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of non-programmable calculator is allowed.

1. Prove that, the nth differences of a rational integral function (polynomial) of the nth degree are constant when the values of the independent variable are at equal intervals. Also, by using the method of separation of symbols, prove that : 15

$$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \ldots\ldots\ldots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}.$$

Or

(*i*) Prove the Newton's forward formula for unequal intervals. 7

(*ii*) Find $\log_{10} 656$, given that, $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$. 8

P.T.O.

2. Prove, the Gauss's central difference forward formula and apply Bessel's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$. 15

Or

(*i*) Explain Euler's modified method to solve the differential equation of the first order : 7

$$\frac{dy}{dx} = f(x, y), \; y(x_0) = y_0.$$

(*ii*) Evaluate : 8

$$\int_0^{\pi/2} \sin x \, dx$$

by using Simpson's $\frac{1}{3}$rd rule.

Given :

$\sin 0 = 0$, $\sin\frac{\pi}{20} = 0.1564$

$\sin\frac{\pi}{10} = 0.3090$, $\sin\frac{3\pi}{20} = 0.4540$

$\sin\frac{\pi}{5} = 0.5878$, $\sin\frac{\pi}{4} = 0.7071$

$\sin\frac{3\pi}{10} = 0.8090$, $\sin\frac{7\pi}{20} = 0.8910$

$\sin\frac{2\pi}{5} = 0.9511$, $\sin\frac{9\pi}{20} = 0.9877$

$\sin\frac{\pi}{2} = 1.0000$.

3. Attempt any *two* of the following : 10

(*i*) Given :

$$u_0 = 3,\ u_1 = 12,\ u_2 = 81,\ u_3 = 200,\ u_4 = 100,\ u_5 = 8.$$

Find $D^5 u_0$.

(*ii*) Prove that the third divided differences with arguments a, b, c, d of the function $\frac{1}{x}$ is equal to $\frac{-1}{abcd}$.

(*iii*) Prove that, $\mu\delta = \frac{1}{2}(\Delta + \nabla)$.

(*iv*) By using Trapezoidal rule, calculate :

$$\int_{-3}^{3} x^4 dx$$

by taking seven equidistant ordinates.