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NA—81—2023

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper–X

(Ring Theory)

(Saturday, 16-12-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define field with an example and prove that every finite integral domain is a field. 15

Or

(*a*) Prove that the intersection of two subrings is a subrings. 8

(*b*) Show that S is an ideal of S + T, where S is any ideal of ring R and T is any subring of R. 7

2. Prove that, the set R [x] of all polynomials over an arbitrary ring R is a ring with respect to addition and multiplication of polynomials. 15

P.T.O.

Or

(*a*) If f is a homomorphism of a ring R into a ring R', then prove that : 8

(*i*) $f(0) = 0'$, where 0 is the zero element of the ring R and 0' is the zero element of R'.

(*ii*) $f(-a) = -f(a), \forall a \in R$.

(*b*) Show that every field is a Euclidean ring. 7

3. Attempt any *two* of the following : 10

(*a*) Show that the ring of integers is a Euclidean ring.

(*b*) If F is a field, then prove that its only ideals are (0) are F itself.

(*c*) If R is a ring, then for all

$a, b, c \in R$, prove that :

$a \cdot 0 = 0 \cdot a = 0$ and

$a(-b) = -(ab) = (-a)b$.

(*d*) Show that the set of matrices :

$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subring of the ring 2 × 2 matrices with integral elements.