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NA—59—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper-VI

(Real Analysis-I)

(Wednesday, 13-12-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Attempt the following :

(*i*) Prove that, the set of rational number is not order-complete. 8

(*ii*) If S and T are subsets of real numbers, then show that : 7

1. $S \subseteq T \Rightarrow S' \subseteq T'$

2. $(S \cup T)' = S' \cup T'$

Or

(*b*) Attempt the following :

(*i*) Prove that, every bounded sequence with a unique limit is convergent. 8

P.T.O.

(*ii*) Show that a sequence $\{S_n\}$

where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots\cdots + \frac{1}{n}, n \in N$ cannot converge. 7

2. (*a*) Attempt the following :

(*i*) If $\lim_{n\to\infty} a_n = l$, then prove that :

$$\lim_{n\to\infty}\left(\frac{a_1 + a_2 + \cdots\cdots + a_n}{n}\right) = l.$$ 8

(*ii*) Show that the sequence $\{b_n\}$ where 7

$$b_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots\cdots + \frac{1}{(2n)^2}$$ converges to zero.

Or

(*b*) Attempt the following :

(*i*) If Σu_n is positive term series such that $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = 1$, 8

then prove that the series :

(1) converges if $l > 1$

(2) diverges if $l < 1$

(3) test fails if $l = 1$

(*ii*) Test the convergence of the series $\sum \frac{n^2-1}{n^2+1}.x^n, x > 0$. 7

3. Attempt any *two* of the following : 10

(*a*) Prove that a sequence cannot converge to more than one limit.

(*b*) If $\lim_{n\to\infty} a_n = a$ and $a_n \geq 0$ for all n, then prove that $a \geq 0$.

(*c*) Test the convergence of the series $\sum \frac{1}{n^{1+\frac{1}{n}}}$.

(*d*) State the Cauchy's root test and the D'Alembert's ratio test.