

This question paper contains 3 printed pages]

NA—61—2023

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (First Year) (Second Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper—III

(Calculus—II (Integral Calculus))

(Thursday, 14-12-2023)

Time : 10.00 a.m. to 12.00 noon

Time—2 Hours

Maximum Marks—40

N.B. :— (i) Attempt *all* questions.

(ii) Figures to the right indicate full marks.

1. Integrate $\frac{1}{\sqrt{ax^2 + bx + c}}$: 15

(i) When a is positive and

(ii) When a is negative ?

Or

(a) Prove that :

8

$$\int_0^{\infty} e^{-x^{1/n}} dx = n\sqrt{n}.$$

P.T.O.

WT

(2)

NA—61—2023

(b) Prove that :

7

$$\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$$

2. Find reduction formula for :

15

$$\int \frac{1}{(y^2 + k^2)^n} dy.$$

Or

(a) Prove that reduction formula :

8

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

(b) Prove that :

7

$$\sqrt{\frac{1}{2}} = \sqrt{\pi}.$$

3. Attempt any *two* of the following :

5 each

(a) Evaluate :

$$\int_0^a \int_0^b (x^2 + y^2) \, dx \, dy.$$

(b) Prove that :

$$\int_{-a}^a f(x) \, dx = 0, \text{ if } f \text{ is odd function of } x.$$

$$= 2 \int_0^a f(x) \, dx, \text{ if } f \text{ is even function of } x.$$

WT

(3)

NA—61—2023

(c) Integrate :

$$\frac{2x - 3}{(x - 1)\sqrt{x + 2}}.$$

(d) Evaluate :

$$\int \frac{x^5}{1 + x^2} dx.$$

JO—61—2023

3