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NA—54—2023

FACULTY OF SCIENCE/ARTS

B.Sc. (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper–I

(Calculus)

(Wednesday, 13-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Find the *n*th derivative of $y = e^{ax} \sin(bx + c)$. Also find the *n*th derivative of $y = \cos^4 x$. 15

Or

(*a*) Find the equations of the tangent and the normal at any point (x, y) of the curve : 8

$$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$$

(*b*) Let $f(x)$ be a function of x. Let this function is to be expanded in ascending powers of x and let the expansion be differentiable term-by-term any number of times, then prove that : 7

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \text{..........}$$

$$\text{.........} + \frac{x^n}{n!} f^{(n)}(0) + \text{............}$$

P.T.O.

2. If $z = f(x, y)$ is a homogeneous function of x, y of degree n, then prove that :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \cdot \partial y} + y^2 \cdot \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

Also show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$$

if $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. 15

Or

(*a*) State and prove Lagranges mean value theorem. 8

(*b*) If in the Cauchy's mean value theorem $f(x) = e^x$ and $f(x) = e^{-x}$, then show that c is arithmetic mean between a and b. 7

3. Attempt any *two* of the following : 5 each

(*a*) Prove that $\cosh^2 x - \sinh^2 x = 1$.

(*b*) Expand $\cos x$ by Maclaurin's series

(*c*) If $f(x) = \tan^{-1} x$, $U < x < v$, then show that $\frac{V-U}{1+V^2} < \tan^{-1} V - \tan^{-1} U < \frac{V-U}{1+U^2}, 0 < U < V$.

(*d*) Find the third order partial derivatives of $U = e^{xyz}$.