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NA—70—2023

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Course)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(Friday, 15-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

1. (A) (*i*) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| \neq 0$ i.e. A is non-singular. 8

(*ii*) Find the inverse of the matrix : 7

$$A = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}.$$

Or

(B) (*i*) Define the following : 8

(*a*) Minor of order K of a matrix.

(*b*) Rank of a matrix.

(*c*) Row equivalent matrix.

(*d*) Column rank of a matrix.

P.T.O.

(*ii*) Find a row echelon matrix which is row equivalent to : 7

$$A = \begin{bmatrix} 0 & 0 & -2 & 3 & 1 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -3 & 1 & 2 \\ 4 & 8 & 2 & 3 & 5 \end{bmatrix}.$$

2. (A) Define characteristic roots and characteristic vectors. Prove that a characteristic vector X of a matrix A cannot correspond to more than one characteristic roots of A. 8

(B) For what values of λ, μ the system of equations : 7

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

has :

(*i*) no solution

(*ii*) a unique solution

(*iii*) an infinite number of solutions.

Or

Express $\sin^n \theta$ in a series of cosines or sines of multiples of θ according as n is an even or odd integer.

Expand $\sin^6 \theta$ in a series of cosines of multiples of θ and $\sin^7 \theta$ in a series of sines of multiple of θ. 15

3. Attempt any *two* of the following : 5 each

(*a*) If A and B are two symmetric matrices of the same order, show that AB is symmetric if and only if AB = BA.

(*b*) Solve the equations :

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$$

$$2x_1 + x_3 - x_4 = 0$$

(*c*) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & 4 & 2 \\ 1 & -3 & 6 & 2 \end{bmatrix}$$

by minor method.

(*d*) Separate into real and imaginary parts of the expression $\cosh(\alpha + i\beta)$.